

THE CONCRETE TO ABSTRACT
MATHEMATICAL INSTRUCTIONAL SEQUENCE
WITH LEARNING DISABLED STUDENTS

BY

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by

Susan Kay Peterson

This work is dedicated to the
University of Florida special education faculty
in appreciation for their many contributions
to my professional growth and development.

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The purpose of this study was to compare the acquisition of an initial place value skill when presented in a concrete, semiconcrete, abstract teaching sequence to acquisition of the same skill when presented abstractly. Measures across time (i.e., posttest, maintenance, and retention) and generalization were included. The subjects in this investigation were learning disabled elementary and middle school students enrolled in special education classrooms.

The study consisted of four phases including a pilot study, teacher training, implementation, and posttreatment. Students were randomly assigned to the experimental and control groups. A 2X3 mixed design with one between (treatment) and one within (performance over

time) group factor was used. A multivariate analysis of variance was computed to determine whether any significant differences existed among the levels of the experimental treatments. Due to obtained significance ($p < .05$), two follow-up univariate analyses of variance were computed. A significant main effect for the instructional treatment variable was found on the acquisition measure. No significant difference was found on the generalization measure. Students who were taught place value using the concrete to abstract teaching sequence performed significantly better on three posttest measures than students who were taught the same skill abstractly. These results have practical implications for classroom instruction and future research.

CHAPTER I INTRODUCTION

Public concern regarding the quality of basic skills instruction in public schools has increased attention on mathematics instruction (Trafton, 1984). General agreement exists regarding the relationship between quality mathematics instruction and student achievement. Isolating the teaching components that constitute quality, however, has been a challenge. Members of the National Council of Teachers of Mathematics (NCTM) discuss the importance of continuing the search for quality instruction. Improving the effectiveness and efficiency of mathematics teaching is stressed (NCTM, 1980). Moreover, questions involving content skill hierarchies, retention, and generalization of arithmetic skills still need to be answered.

The current educational emphasis on acquisition of basic skills, minimum competency testing (McCarthy, 1983), and progression through basal math programs has affected mathematics instruction. Teachers feel compelled to cover prespecified amounts of material at a rapid pace. Correct computational answers are the primary measure for student success in mathematics and are frequently used to determine readiness for teaching a new skill.

Davis (1983) cautions against an overemphasis on computation at the expense of teaching important mathematical concepts. He notes that the teaching of concepts is especially important to students with learning problems. Students who lack conceptual understanding of mathematical skills tend to rely on memorization of algorithm steps to solve arithmetic problems. Clearly, this increases the likelihood of computational errors. Consider, for example, the student who is asked to demonstrate the subtraction problem 23 minus 17 using place value sticks. If the student has mastered the concept of subtraction with regrouping, he or she will take a ten-bundle and add it to the three single sticks making 13 ones and then proceed to remove the 1 ten and 7 ones represented in 17. A student who has not mastered the regrouping concept will typically claim "it cannot be done." The same student may be able to solve the problem verbally or with paper and pencil due to algorithm memorization. When the child is unable to demonstrate the problem concretely, conceptual understanding has not been mastered. Copeland (1979), Reisman (1982), and Underhill, Uprichard, and Heddens (1980) suggest that acquiring an understanding of basic computational concepts will influence future success when learning higher level math skills. Moreover, Davis (1983) reports the extent of a student's conceptual knowledge is probably a strong indicator of performance in

class, on tests, and in applying mathematics to solve problems. Therefore, teaching fundamental mathematical concepts appears critical to successful instructional programs.

Mathematical skills and concepts are typically taught sequentially. Initial instruction involves simple number relationships and then progresses to more complex tasks. Underhill et al. (1980) suggest that as students progress sequentially through arrangements of mathematical tasks they transfer the learning of skills from one level to the next higher level. Acquisition of new skills is, therefore, dependent upon previous learning.

Hierarchical approaches to mathematics are used in most classrooms. Some teachers also incorporate a mathematical concept sequence for skills taught within the hierarchies. It is generally agreed that the concept sequence progresses from concrete to semiconcrete to abstract learning experiences (Beattie, 1986). Instruction using the concept sequence begins with the manipulation of objects, then uses pictures or tallies, and finally involves the abstract symbols (i.e., numbers) in isolation.

Although the manipulation of objects, uses of pictures or tallies, and the use of abstract symbols teaching sequence has widespread theoretical acceptance, little research exists to document its effectiveness. Professional opinion rather than empirical studies supports

the use of the concrete to abstract continuum. Comparisons between teaching skills abstractly and teaching skills using the concrete to abstract sequence have not been reported. Generalization and retention measures resulting from this teaching sequence are also missing from the literature.

Statement of the Problem

The problem investigated in this study was the effectiveness of teaching learning disabled students place value through a conceptual sequence including three levels of understanding (i.e., concrete, semiconcrete, and abstract) compared to the effectiveness of teaching the same skill at the abstract level without manipulatives or pictorial representations. The experimental questions were as follows:

1. Is the concrete to abstract instructional sequence more effective than the abstract only approach in teaching the acquisition of place value skills?
2. Is the concrete to abstract instructional sequence more effective than the abstract only approach in maintenance of the place value skill?
3. Is the concrete to abstract instructional sequence more effective than the abstract only approach for promoting retention of the place value skill?

4. Is the concrete to abstract instructional sequence more effective than the abstract only approach in promoting generalization to higher level place value skills?

The problem this study examined is important for several reasons. First, the study adds to the sparse research data on teaching learning disabled students mathematical skills using a concrete, semiconcrete, and abstract instructional sequence. Second, this study provides teachers with information regarding expected skill retention when implementing instruction abstractly and when implementing the concrete to abstract sequence to learning disabled students. Third, it contributes to data related to the relationship between this mathematical intervention and skill generalization among students experiencing learning difficulties. Finally, the study provides data concerning effective strategies for promoting concept understanding among students with learning difficulties.

Rationale

Psychologists and educators have been investigating mathematical learning for many years (Buswell & Judd, 1925). For the most part, the findings have been an eclectic collection of unrelated studies with sometimes conflicting results (Lesh & Landau, 1983). Moreover, Bartel (1982) states that learning difficulties in

arithmetic have received less attention than other academic areas. Research efforts within the last few years, however, reflect improvements in the study of mathematics. A cohesive body of knowledge is beginning to develop (Lesh & Landau, 1983).

Undoubtedly, additional research will assist teachers who are continuously faced with children experiencing mathematical difficulties. There appear to be many reasons students have problems with mathematics. Ashlock (1986) reports that students frequently do not have prerequisite understandings and skills needed to learn new ideas and procedures. He further states that teachers who introduce paper-and-pencil procedures to students who still need to work problems with concrete aids are encouraging students to memorize a complex sequence of mechanical acts. Such memorization lends itself to faulty algorithms and frustration for the student and teacher.

Reisman (1982) concurs with Ashlock's position and further suggests that students must develop a basic understanding of mathematical relationships before they can succeed at arithmetic computation. Gaps in mathematical foundations occur when relations underlying computational algorithms are not fully understood.

Thus, the development of meaning and understanding within the learner's capacity to perform has become a recent goal for school mathematics (Underhill et al.,

1980). The challenge this represents for teachers elicits numerous questions. What teaching techniques enable students to understand concepts underlying arithmetic computations? How much practice, if any, is required at preabstract teaching levels? Will concept development and an emphasis on meaning improve generalization and retention? Empirical research, such as this study, must be employed to add to the existing data base related to the instruction of mathematics. Information derived from systematic examinations of instructional procedures can improve teacher training at the preservice and inservice levels. Improved teacher effectiveness results in an increased likelihood that students will experience successful mathematical endeavors throughout their school careers.

Definitions

The following section presents terms and their definitions as used in this study. Terms selected for inclusion are critical to understanding implementation procedures and observed results.

Concrete learning experience.

A concrete learning experience is one that helps the learner relate manipulative processes and computational processes. The learner focuses on both learner-manipulated objects and the symbolic processes which accompany, correlate to, and describe the manipulations. (Underhill et al., 1980, p. 28)

Semiconcrete learning experience.

A semiconcrete learning experience is one that is visual, but is not tactile or kinesthetic as is the concrete learning experience. Examples include drawing pictures or diagrams; watching slides, filmstrips, or movies; watching a demonstration with manipulatives; and looking at pictures and diagrams in textbooks or worksheets. The focus is on building associations between visual modeling and symbolic processes. (Underhill et al., 1980, p. 29)

Abstract learning experience. "An abstract learning experience is one in which the learner uses no manipulatives or visual process reinforcers to build associations" (Underhill et al., 1980, p. 30). Only written symbols (i.e., numbers) are used.

Celeration. Celeration is the "basic unit of measurement of behavior change. Change in frequency per unit time; movements/minute/week" (Pennypacker, Koenig, & Lindsley, 1972, p. 111).

Criterion. The criterion is a specified value to be obtained during academic performance before proceeding to the next teaching task (i.e., a performance standard).

Direct instruction.

Direct instruction refers to high levels of student engagement within academically focused, teacher-directed classrooms using sequenced, structured materials. . . . [D]irect instruction refers to teaching activities focused on academic matters where goals are clear to students; time allocated for instruction is sufficient and continuous; content coverage is extensive; student performance is monitored; questions are

at a low cognitive level and produce many correct responses; and feedback to students is immediate and academically oriented. In direct instruction, the teacher controls instructional goals, chooses material appropriate for the student's ability level, and paces the instructional episode. Interaction is characterized as structured, but not authoritarian; rather, learning takes place in a convivial academic atmosphere. (Rosenshine, 1977, p. 17)

Generalization. "Generalization is the occurrence of a discriminated response in the presence of an unknown or novel stimulus" (Miller, 1975, p. 179).

Maintenance. Maintenance is an instructional procedure in which periodic checking occurs to ensure the skill is still performed at criterion. For the purposes of this study, maintenance is measured 1 week after instruction.

Place value. Place value is the value assigned to a digit based on its position in the numeral (Reisman, 1981).

Precision teaching. Precision teaching is a comprehensive instructional system for accelerating learning and maintaining high proficiency based on direct and continuous measurement procedures.

Probe. A probe is an "ancillary measurement system for applied behavior analysis and precision-teaching systems" (Cawley, 1984, p. 70).

Retention. Retention is the ability to remember information previously learned. For the purposes of this study, retention is measured 3 weeks after instruction.

Delimitations

The scope of this study was delimited in three ways. First, the study was restricted geographically to Alachua and Pinellas counties within the state of Florida. Second, only learning disabled students were included. Third, only subjects in public elementary and middle school grades were considered for the study.

Limitations

Since this study included only learning disabled elementary and middle school students, the findings should not be generalized to other handicapped, nonhandicapped, or high school students. Moreover, the results of this study cannot be generalized to other mathematical skills without replication. Caution should be exercised in extrapolating results of the study to students who live outside Alachua and Pinellas counties.

Summary

The current trend toward increased research involving specific mathematical interventions is greatly needed. Lack of attention given to the development of underlying mathematical concepts has hindered students' abilities to succeed at higher computational levels. It was the intent of this study to contribute information regarding the

effectiveness of teaching place value concepts through a concrete to abstract sequence as compared to abstract instruction without concrete or semiconcrete experiences. Specifically, acquisition, maintenance, retention, and generalization were analyzed. The results of this study have direct and immediate implications for teachers of mathematics.

A review of literature relevant to this study is presented in Chapter II. Methodology used for implementation is discussed in Chapter III. The results and their implications are reported in Chapters IV and V.

CHAPTER II REVIEW OF RELATED LITERATURE

The purpose of this chapter is to summarize and analyze existing professional literature related to concrete, semiconcrete, and abstract instructional procedures for teaching mathematical skills. First, inclusion criteria for research reviewed are stated. Second, a historical summary from the literature supporting the use of concrete, semiconcrete, and abstract mathematical instruction is presented. Third, a description and a critique of research involving comparisons of concrete, semiconcrete, and abstract teaching components are presented. Fourth, a study involving the concrete to abstract components in sequence is examined.

Inclusion Criteria

The following criteria were used to select data-based research reports for this literature review. The reports needed to have

1. experimental question(s) addressing the effectiveness of concrete, semiconcrete, and/or abstract teaching procedures used in isolation or in varying teaching sequences;

2. dependent variable(s) which could be classified as academic behaviors; and
3. complete research report data, including description of subjects, methodology, and results.

Sources used for locating relevant literature included card catalogs at the University of Florida libraries; Current Index of Journals in Education; Educational Resources Information Center; the interlibrary loan services; and the reference sections of related articles, chapters, and books. Five studies were located that compared the effectiveness of concrete vs. semiconcrete vs. abstract teaching strategies. One study was located that compared the effectiveness of the concrete to abstract teaching sequence to the effectiveness of the semiconcrete to abstract teaching sequence.

Historical Perspective

The idea that mathematical learning progresses through three levels of abstraction has evolved since the beginning of the nineteenth century (Burton, 1984; Suydam & Higgins, 1977). Pestalozzi is credited with emphasizing the need for object-teaching. Moreover, he suggests that children must experiment in the concrete, before applying abstract rules or exercises (Cajori, 1896, pp. 211-212). By the 1930s, many mathematicians accepted the premise that

students should progress through three stages involving objects first, then pictures, and finally symbols.

Bruner (1966) and Piaget and Inhelder (1958) identify these stages as enactive, iconic, and symbolic. These authors contend that students first learn mathematics through manipulating concrete objects in an enactive sense. When this is completed, the students can learn through pictures or representations of objects in an iconic manner and finally through manipulating abstract symbols (Suydam & Higgins, 1977).

Underhill et al. (1980) identify the sequence components as concrete, semiconcrete, and abstract. These investigators report that during initial skill learning, the student needs to concentrate on both the manipulated objects and the symbolic processes that describe the manipulations. Moreover, Underhill et al. suggest the goal of mathematical curricular activities is to build stepwise associations through practice experiences so that the learner will not always need concrete manipulatives, but will eventually be able to think about the set of experiences (iconic imagery). Because the symbols used relate to real-world experiences, mathematical computation will be meaningful to the learner.

More recently, Heddens (1984) suggests dividing the semiconcrete stage into two components, semiconcrete and semiabstract. Heddens (1986) defines the semiconcrete

level as a representation of a real situation (e.g., pictures of real objects). The semiabstract level involves using symbols to represent concrete objects. For example, tallies might be used instead of pictures.

Although general acceptance exists regarding the idea that learners understand mathematics at different cognitive levels and that these levels follow a developmental sequence, questions still remain. Is this sequence necessary for all mathematical skills? Is this sequence required for some learners but not others? Does such a teaching sequence produce acceptable generalization and retention abilities? Do concrete, semiconcrete, and abstract teaching procedures presented separately have differing effects on achievement? Answers to some of these questions are beginning to emerge in the professional literature.

Comparisons of Concrete, Semiconcrete, and Abstract Teaching Procedures

Research on teaching mathematical skills using concrete, semiconcrete, and abstract procedures has emphasized comparing the effectiveness of these three modes of presentation used in isolation. Typically the studies compare concrete instruction to either semiconcrete or abstract instruction (Armstrong, 1972; Fennema, 1972; Prigge, 1978). Scott and Neufeld (1976) and Smith, Szabo, and Trueblood (1980) compared all three modes (i.e.,

concrete vs. semiconcrete vs. abstract) of presentation (see Table 1).

The studies included in this review are discussed and critiqued in terms of their research designs, subject characteristics, measurement methods, and experimental procedures. Finally, study results are reported and summarized.

Research Design

A group research design was used in all reviewed studies to determine the effectiveness of the intervention. Random assignment and pre- and posttest measures were used in each design to facilitate experimental control and provide increased opportunities for the investigators to observe treatment effects.

The simplest research design involved two treatment groups, manipulative (characterized by enactive modes of representation of the mathematical concepts) and nonmanipulative (characterized by symbolic and/or iconic representational modes). These groups were compared to determine effect on concept acquisition (Armstrong, 1972). Smith et al. (1980) used a similar design. Included in their design, however, was a third treatment group involving graphics (semiconcrete). Scott and Neufeld (1976) extended this basic design format to include a control group. Students receiving abstract modes of instruction served as a control group for two treatment

Table 1
Review of Mathematical Studies Using Concrete,
Semiconcrete, and/or Abstract Instruction

Author(s)	Subjects	Intervention	Skill(s)
Armstrong (1972)	1. 20 TMR students	manipulative vs. nonmani- pulative	1. numeral-quantity association
	2. 67 EMR students		2. conversion of quantity
			3. numeral identi- fication
			4. counting

Results: The manipulative mode was significantly better than nonmanipulative mode on numeral-quantity association. No significant differences were found on the other three skills.

Fënnema (1972)	95 second graders	concrete vs. symbolic	premultiplication concepts
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Results: No significant difference was found on direct recall. The symbolic treatment was significantly better than concrete treatment on a transfer test.

Scott & Neufeld (1976)	145 second graders	concrete vs. pictorial	multiplication
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Results: No significant difference was found between concrete and pictorial modes of instruction.

Prigge (1978)	146 third graders	no manipula- tives vs. two- dimensional manipulatives vs. three-di- mensional aids	geometric concepts (i.e., point, line, line segment, triangle, square, etc.)
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Results: The three-dimensional manipulatives were significantly better than no manipulatives and two-dimensional manipulatives when used with low achieving students.

Table 1--continued.

Author(s)	Subjects	Intervention	Skill(s)
Smith, Szabo, & Trueblood (1980)	66 first and second graders	manipulative vs. graphic vs. abstract	relational terms (i.e., longer, shorter, same length as)
Results: Manipulative treatment was significantly better than graphic; no significant difference between the manipulative and abstract was found.			

groups: one receiving pictorial instruction and the other receiving manipulative instruction. The remaining studies used multiple groups to test treatment effects. Fennema (1972) used eight groups in her study to compare symbolic and concrete instruction. Prigge (1978) used six groups to compare three treatments, two of which involved manipulatives. He compared the effectiveness of two-dimensional manipulatives to three-dimensional manipulatives to abstract symbols for mathematical instruction. A 3X3 factorial design was used.

Subject Characteristics

The number of subjects in the reviewed studies ranged from 20 (Armstrong, 1972) to 146 (Prigge, 1978). A higher incidence of second graders was noted (Fennema, 1972; Scott & Neufeld, 1976; Smith et al. 1980). One study (Prigge, 1978) used third graders. The second and third grade studies were implemented in regular education classrooms.

Armstrong (1972) conducted two studies. Grade levels were not reported in either study. His population consisted of special education students. Twenty trainably mentally retarded students with an average mental age of 2.4 were used in the first study. Sixty-seven educably mentally retarded students with a mental age ranging from 5.8 to 11.9 were used in the second study.

Measurement Methods

All studies included in this review used pretest and posttest measures to establish research control and strengthen reported results. The type and purpose of selected pre- and posttest measures varied extensively among the studies. Prigge (1978) used the Iowa Test of Basic Skills to assess pretreatment mathematical skill acquisition. No other study reviewed mentions a commonly used standardized test for pretest measures. Fennema (1972) and Smith et al. (1980) discuss specific skills measured during pretesting. The former tested for length conservation while the latter tested for knowledge of cardinal numbers of sets (1-10), equivalent sets, numerals, and addition of one-digit numbers. These investigators, however, do not report the specific instruments used for testing these skills. Armstrong (1972) simply states that an individualized test of the major mathematical concepts taught was administered. Even less information is offered in the Scott and Neufeld (1976) study. Pretest scores were used as covariates when analyzing the data.

More information was given regarding posttreatment measures. The investigators in each study constructed the instruments used for posttesting. Several authors report the validity and reliability of their postmeasurement instruments (Fennema, 1972; Scott & Neufeld, 1976; Smith et al. 1980). Fennema presents reliability coefficients of

.91, .92, and .93 for a test of recall and two tests of transfer, respectively. Scott and Neufeld established their instrument's reliability at .94. Lower figures are reported in the study of Smith et al. They are .58, .78, and .45 for three criterion instruments used. Content validity was established via test construction in the studies of Fennema and Smith et al., while four experts' judgment was used in the Scott and Neufeld study.

All studies reviewed included mathematical skill acquisition among the dependent measures. One investigation also measured affective student responses (Scott & Neufeld, 1976). Fennema (1972) and Prigge (1978) included a generalization and retention measure in their investigations, respectively. Mathematical skills taught in the studies differed according to student needs. Multiplication skills were taught in two studies (Fennema, 1972; Scott & Neufeld, 1976). Measurement skills were taught in the research of Smith et al.; geometric concepts were taught by Prigge; and Armstrong's (1972) experiments presented readiness skills involving set formation, numeral quantities, and counting.

Experimental Procedures

The treatment conditions range in time from 4 days (Smith et al., 1980) to 20 days (Armstrong, 1972; Scott & Neufeld, 1976) assuming one lesson was presented per day. Four studies (Armstrong, 1972; Fennema, 1972; Prigge, 1978;

Scott & Neufeld, 1976) report the use of systematic, controlled lessons. Smith et al. (1980) do not mention attempts to keep lesson procedures constant.

Materials. All studies reviewed involve the use of concrete manipulatives. The specific objects used vary across studies. Fennema (1972) used Cuisenaire rods, a commercially made concrete mathematical material. Manipulatives used in the other studies include paper clips, rulers, clay, cubes, geoboards, paper folding, tetrahedrons, parallelepiped, plastic animals, and assorted solids. The type of manipulatives used in the study of Scott and Neufeld (1976) was not reported.

Instructional medium. Teachers are used in four studies (Fennema, 1972; Prigge, 1978; Scott & Neufeld, 1976; Smith et al., 1980) to provide the instruction. In one study, autoinstructional programs provide instruction using slides and tapes (Armstrong, 1972). Equipment, in this case, was used to control for teacher effect. In Armstrong's second study, with trainably mentally retarded students, the machines were not used due to student behavioral difficulties. Teacher effect was controlled by rotating teachers through each of the two intervention conditions.

Experimental constants. Attempts to keep the instructional techniques and educational materials constant throughout the investigation periods were reported in all

reviewed studies. Several authors mention constant instructional time limitations, which range from 30 to 60 minutes a day (Prigge, 1978; Scott & Neufeld, 1976; Smith et al., 1980).

Results

Four studies (Armstrong, 1972; Fennema, 1972; Prigge, 1978; Smith et al., 1980) include components that provide a comparison of concrete to abstract modes of instruction. Results from this research are inconsistent. Concrete instruction was significantly better in Prigge's research and in Armstrong's research. Fennema and Smith et al., however, found no significant differences between the two.

Studies comparing concrete to semiconcrete instruction also reveal mixed results. Armstrong (1972) and Smith et al. (1980) state concrete instructional modes are significantly more effective than semiconcrete modes. Scott and Neufeld (1976), however, report no significant difference between the two.

The results presented in the studies reviewed are inconclusive. Trends are not evident. It is apparent that further research is needed to best determine effective modes for mathematical instruction.

Sequences of Concrete, Semiconcrete, and Abstract Teaching Procedures

St. Martin (1975) performed a study that examined the effect of teaching mathematical skills in the concrete to

abstract sequence on skill acquisition and retention. Two types of teaching sequences were compared (i.e., concrete to semiconcrete to abstract vs. semiconcrete to abstract). Moreover, St. Martin wanted to determine if there was a significant difference between initial achievement or retention as a result of the Piagetian level of development (see Table 2).

Table 2
Concrete to Abstract Sequence Study

Author	Subjects	Intervention	Skill
St. Martin (1975)	99 fifth graders	concrete to abstract vs. semiconcrete to abstract	multiplication and division of fractions

Results: No significant difference was found on acquisition or retention between the two instructional sequences.

St. Martin's (1975) study is summarized and critiqued in the following section. The research design, subject characteristics, measurement methods, experimental procedures, and results are discussed. Analysis of St. Martin's study is reported separate from the other five studies in this literature review due to the concrete to abstract sequencing component.

Research Design

St. Martin (1975) used two treatment groups (i.e., concrete to abstract and semiconcrete to abstract). He employed a 2X2 factorial design to compare treatment effects, Piagetian level effects, and their interaction effect on achievement.

Random selection was used to pick a school and four specific classes within the school. Students in the study were randomly assigned to one of the two treatment groups. Also, included in St. Martin's (1975) design were pretests and posttests to measure performance changes. The factorial design was appropriate for answering the specified research questions.

Subject Characteristics

The subjects for St. Martin's (1975) study were four fifth grade classes from the L. J. Alleman Elementary School in Lafayette, Louisiana. All four classes were heterogeneously grouped. The average age for the subjects was 11 years, 1 month. A total of 99 students were included in the study.

Measurement Methods

Three author-constructed tests were used: one covering multiplication of fractions, one covering division of fractions, and one covering both to measure retention. The scores on the initial two instruments were combined. The author's rationale for combining test scores and

combining content on the retention test was unclear. Perhaps logistically such combinations were facilitative in terms of time and effort. It seems, however, that these measurement decisions may have increased the confounding variables in the study.

St. Martin (1975) also used a standardized measure in his study. The Science Research Associates (SRA) Assessment Survey was used as the pretest. According to the author, this instrument was selected because it seemed to correspond to the textbook program used at the school. The inclusion of a pretest did not seem to strengthen the study since it was selected based on similarity to a mathematical textbook that had little relevance to the study.

Experimental Procedures

The instructional phase of the study lasted 27 days including time for administering the achievement instruments. The follow-up retention measure occurred 4 weeks after instruction ended. Each instructional period lasted 55 minutes.

Manipulatives used for concrete instruction included fraction parts made from construction paper, Cuisenaire rods, poker chips, and geoboards. The medium for providing the instruction was St. Martin, the primary investigator. The teachers of the four classes served as assistants. Teacher effect was controlled by having the same individual

providing both treatments. The implementation, however, may have been biased since the primary investigator planned, implemented, and interpreted the results of the study. Controlling for teacher effect via teaching scripts may have increased the objectivity of the study.

Results

St. Martin's (1975) results suggest that the concrete to abstract and semiconcrete to abstract teaching sequences are equally effective for initial achievement and retention when used with fifth grade students at both Piagetian Level III and IV. Level IV students did retain information better than Level III. Informal observations showed that lower level students responded well to concrete materials.

In summary, St. Martin's (1975) study did not produce any conclusive evidence with regard to the interaction of the two teaching sequences with Piagetian development levels. Therefore, additional research in this area is needed to test the effectiveness of various teaching sequences.

Summary

The topic explored in these research studies is of paramount importance to mathematical educators. The search for instructional modes that result in efficient and effective mathematical learning is very important. The

lack of replicable findings among current research is reason for concern.

Historically, mathematical educators have promoted the premise that new skills should be taught concretely, pictorially, and then symbolically. An examination of the literature, however, reveals an obvious lack of data to support this generally accepted premise. Although literature exists documenting extensive mathematical problems among learning disabled students (Carpenter, 1985), the concrete to abstract theory has not been empirically tested with handicapped students and research available involving nonhandicapped students is limited both in scope and context.

The results of existing studies are inconsistent. Therefore, sound conclusions are lacking. Clearly, further research in the concrete to abstract instructional methodology is needed.

CHAPTER III METHOD

The research method in this investigation was designed to compare the use of a three-level (i.e., concrete to abstract) teaching sequence to an abstract level teaching procedure. Effect on initial acquisition, maintenance, retention, and generalization of a place value skill was measured. The research methodology including descriptions of the subjects, null hypotheses, instrumentation, materials, procedures, and experimental design and analysis is outlined in Chapter III.

Subjects

The subjects for this study were learning disabled elementary and middle school students. There were 20 males and 4 females ranging in age from 8 to 13. All subjects received mathematical instruction in a special education classroom located in either Alachua or Pinellas counties. Of the 24 subjects included, 19 were in self-contained classrooms for the learning disabled, 4 were temporarily placed in a self-contained diagnostic classroom, and 1 was in a resource room setting. The four students placed in the diagnostic classroom returned to their home school settings after 6 weeks. One returned to a self-contained

learning disabilities class and three returned to resource room settings.

Additional subject characteristics are summarized in Table 3 per the recommendation of Smith et al. (1984). These authors suggest including sex, age, race, socioeconomic status (SES), intelligence quotient (IQ) scores, and achievement scores when describing learning disabled subjects.

All subjects participated in a two-part screening procedure to determine eligibility for the study. Screening occurred to ensure that the students participating did not have preexisting knowledge regarding the acquisition and generalization of expected target skills. To be included students had to score 70% or lower on both parts (i.e., acquisition and generalization) of the screening instrument. Subjects who qualified for inclusion in the study were randomly assigned to the treatment and control groups.

The subjects in the control and treatment groups had very little knowledge of the assessed place value skills. The mean performance of the control group on the screening device was 22.92% for acquisition and 4.58% for generalization. The mean performance of the treatment group was 28.75% for acquisition and 1.67% for generalization. Neither group could successfully identify

Table 3
Summary of Subject Characteristics

Experimental Group	Control Group	Experimental Group	Control Group
Numbers: male 10 female 2 total 12	Numbers: male 10 female 2 total 12	IQ: mean 89.0 range 73-103 test(s) WISC-R used K-ABC	IQ: mean 91.5 range 80-119 test(s) WISC-R used K-ABC
Age: mean 10.5 range 9-13	Age: mean 10.3 range 8-12	Math achievement: mean 74.45 range 64-88 test WRAT-R used K-TEA	Math achievement: mean 70.45 range 60-83 test WRAT-R used
Race: Anglo 9 Hispanic 0 Black 3 Other 0	Race: Anglo 7 Hispanic 0 Black 5 Other 0	Reading achievement: mean 67.27 range 48-81 test WRAT-R used K-TEA	Reading achievement: mean 65.36 range 55-92 test WRAT-R used
SES: high 4 middle 1 low 7	SES: high 2 middle 0 low 10	Spelling achievement: mean 63.36 range 51-78 test WRAT-R used K-TEA	Spelling achievement: mean 69.91 range 54-94 test WRAT-R used

ones and tens in double digit, three digit, or four digit numbers prior to instruction.

Hypotheses

Several variables were examined in this study to ascertain possible differences between the three-level sequential procedure and the one-level teaching procedure. The following null hypotheses address the two treatment conditions related to acquisition and generalization of student performance.

- H1: There will be no significant difference between students receiving three- and one-level instruction in place value acquisition scores obtained immediately following treatment.
- H2: There will be no significant difference between students receiving three- and one-level instruction in place value acquisition scores on a maintenance probe administered the week following instruction.
- H3: There will be no significant difference between students receiving three- and one-level instruction in place value acquisition scores obtained 3 weeks after instruction has ceased.
- H4: There will be no significant difference between students receiving three- and one-level

instruction in place value generalization scores obtained over time.

The .05 level of significance was used as the basis for rejection of a null hypothesis.

Instrumentation

In this study, four teacher-made research instruments were used. Each instrument had two parts: one to measure acquisition and one to measure generalization. Administration occurred to measure place value acquisition and generalization immediately following instruction, maintenance of the skills 1 week later, and retention 3 weeks after instruction.

Teacher-Made Screening Device

The screening device had two parts (see Appendix A). Part one required the subject to write how many ones or tens were found in various two-digit numbers. Part two of the screening device required the student to write the number of tens or ones found in three- and four-digit numbers.

Teacher-Made Posttests

The teacher-made posttests were designed to measure students' understanding of place value. There were 20 items included on part one of the posttest and 20 items on part two. Three alternate forms were designed to measure skill acquisition and generalization immediately following

instruction, the week after instruction, and 3 weeks after instruction (see Appendices B, C, and D). The three alternate forms were randomly assigned to all subjects for each of the repeated measures to ensure instrument reliability.

Materials

Materials used with students receiving concrete instruction included plastic unifix cubes produced by Philograph Publications LTD, place value sticks (i.e., popsicle sticks), and teacher-made place value strips. Materials used with these same students for semiconcrete instruction were Scott, Foresman and Company and Macmillan Publishing Company worksheets. The worksheets selected demonstrate place value with differing modes of representation. Materials used for abstract instruction included Scott, Foresman and Company; Macmillan Publishing Company; and teacher-made worksheets without any pictorial representations (see Appendix E). Additionally, a see-say place value probe sheet was used during instruction. All subjects included in the study had the same sheet (see Appendix F).

Procedure

The procedure for this study was divided into four phases. Phase one was a pilot study designed to test

instructional activities to be used in the concrete to abstract teaching sequence. Phase two was a training period for the teachers involved in the study. Phase three was the instructional implementation lasting nine teaching days. Direct instruction methodology was employed to maximize academic performance. The final phase consisted of posttreatment skill acquisition, maintenance, retention, and generalization measures. The remainder of this section will provide an overview of direct instruction and discuss each procedural phase in detail.

Direct Instruction

The direct instruction methodology was developed from Becker's work on the use of reinforcement to change classroom behaviors and from Engelmann's work in the Bereiter and Engelmann (1966) preschool program (Becker & Carnine, 1981). The direct instruction model "is based on the assumption that every child can achieve well in school if he or she receives adequate instruction; conversely, pupil failure is a direct result of instructional failure" (Nero & Associates, 1976, p. 348).

Typically, direct instruction refers to teaching activities that focus on academic matters with goals that are clear to the students, provide sufficient time for instruction, cover content extensively, monitor student performance, present questions at a low cognitive level to produce many correct responses, and provide immediate and

academically oriented feedback to students (Rosenshine, 1976). The teacher determines the instructional goals, selects appropriate materials for students' functioning levels, and paces the teaching. Interaction is structured, positive, and nonauthoritarian.

Much research has been done to document the effectiveness of direct instruction. In Follow Through data, the direct instruction model showed greater gains in mathematics, reading, spelling, and language than eight other models of teaching (Jewell & Feiler, 1985; Suydam, 1985).

Several individual components of direct instruction have been researched. Findings suggest that teachers who maintain a brisk pace and a high rate of progress through the curriculum produce greater academic gains than teachers who do not (Brophy, 1979; Carnine, 1981). Teachers who provide successful practice at levels of 80% accuracy or higher positively affect the performance of low-achieving students (Rosenshine, 1983; Stevens & Rosenshine, 1981). Finally, teachers who provide immediate teacher feedback following correct and error responses produce greater learning than teachers who do not provide immediate feedback (Rosenshine, 1980).

Due to much data supporting direct instruction, this model was incorporated into the teaching procedures designed for both groups in this study. Thus, any

differences noted between the groups would truly be a reflection of the specific math interventions and not the teaching methodology.

Pilot Study (Phase One)

Subjects. The three male subjects involved in the pilot study were enrolled in the Multidisciplinary Diagnostic and Training Program's (MDTP) classroom housed in Norman Hall on the University of Florida campus in Gainesville, Florida. Each subject attended the MDTP classroom for 5 weeks and then returned to his home school (i.e., Metcalfe and High Springs Elementary Schools in Alachua County and Melrose Elementary in Columbia County). Learning characteristics for each student are summarized in Appendix G.

Design. The primary purpose of the pilot study was to test the implementation of three learning activities at the concrete, semiconcrete, and abstract instructional levels. A multiple baseline (Baer, Wolf, & Risley, 1968), single subject design was used. Single subject designs provide within subject control and are appropriate for documenting the effectiveness of an academic intervention used with several individual students.

Specific to this research, the multiple baseline across three individuals receiving concrete to abstract instruction on place value occurred. The subjects were determined eligible for the study based on a place value

pretest (see Appendix H). Criterion for study participation was a score of 70% or less. Baseline data were collected simultaneously across all three subjects. When the baseline data exhibited acceptable stability in trend, the concrete to abstract intervention was applied to the first subject. When criterion-level performance was attained, the intervention was applied to the second subject. The same procedure was used for application of the intervention to the third subject.

Three phases were included in the pilot study design: a baseline phase, a treatment phase, and a posttreatment phase (see Figure 1). Baseline and treatment phases took place in the University of Florida's MDTP classroom. The posttreatment phase took place in the students' home schools.

Baseline phase. During the baseline phase, 1-minute precision teaching timings were administered to each subject on a daily basis. The same instructor met individually with each subject during the same work period each day. The same teacher dialogue was used to present the probe on each occasion (i.e., "I want you to tell me how many ones or tens the underlined number represents"). Appendix I illustrates the precision teaching probe used throughout the pilot study. Teacher feedback regarding student performance was withheld. The students'

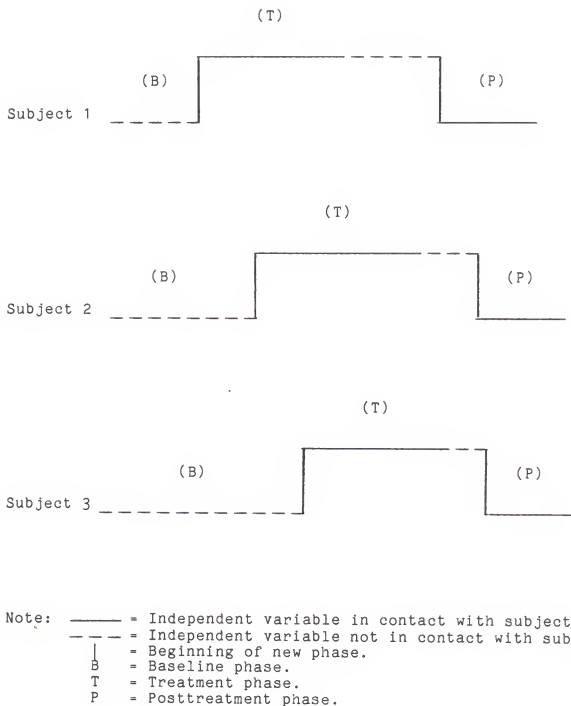


Figure 1
Diagram of Pilot Study Experimental Conditions

performance data were entered on AC-4 Standard Behavior Charts (Berquam, 1979). The charts, however, were not discussed with the students. Baseline data were gathered for a minimum of 3 days (Tawney & Gast, 1984).

Treatment phase. During the treatment phase the three subjects were taught place value using a concrete, semiconcrete, and abstract teaching package. A minimum of three individual activities were presented to each subject for concrete, semiconcrete, and abstract instruction. All activities were presented using a direct instruction model. Steps included in the model were

1. Provide an advance organizer.
2. Demonstrate and model the skill.
3. Provide guided practice.
4. Provide independent practice.

Instruction during the treatment phase was structured the same for all subjects. A teacher script was used and instructional time was limited to 15 minutes per day. At the completion of each 15-minute instructional period, a 1-minute precision teaching timing was administered to the student using the same baseline probe sheet. Correct and incorrect responses were charted on the AC-4 Standard Behavior Charts (Berquam, 1979).

Students progressed through the nine activities at their own rate. Criterion checks were made to ensure student comprehension before moving to a new activity. The

duration of the treatment phase ranged from 9 to 15 days for the three subjects. The variability in number of instructional days resulted from individual differences in ability to meet the criterion set for acceptable performance on the designated tasks.

Posttreatment phase. Immediately following the last instructional activity each student was given a posttest identical to the pretest. Direct instruction on the mathematical skill was discontinued. The same precision teaching probe used during Phases A and B was administered periodically for 2 weeks to serve as a maintenance check for the subjects after completing the treatment intervention. Next, maintenance checks were discontinued for a week. Finally, the primary investigator introduced a retention probe (see Appendix J). An alternate form of the posttest was given along with another 1-minute timing.

Results. All three students made dramatic gains on the criterion-referenced posttest measures. More importantly each student retained the newly acquired skill. Table 4 displays the percentage scores for the three measures. Daily 1-minute timings on student skill acquisition were recorded throughout the study (see Figure 2). During the baseline phase all three students had more error responses than correct responses. During the treatment phase correct responses increased while error responses decreased. All three subjects demonstrated

Table 4
Place Value Pilot Study Percentage Scores

Subject	Pretest	Posttest	Retention
1	0%	80%	80%
2	0%	90%	80%
3	40%	80%	80%

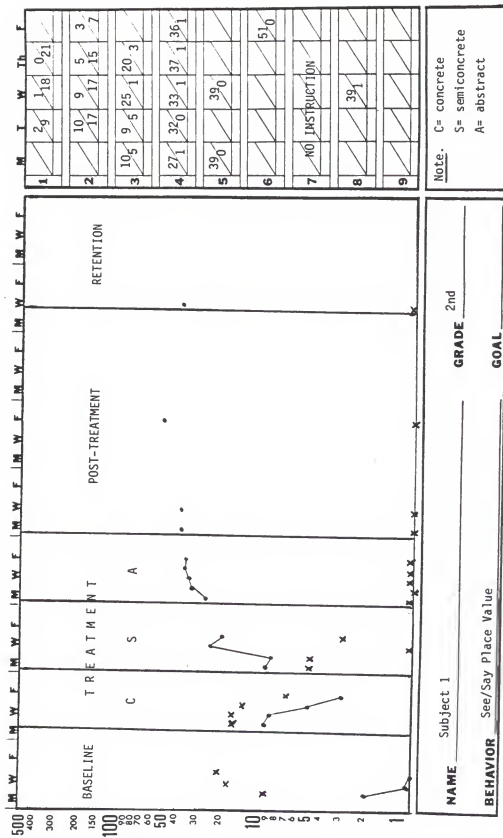


Figure 2
Pilot Study Timings

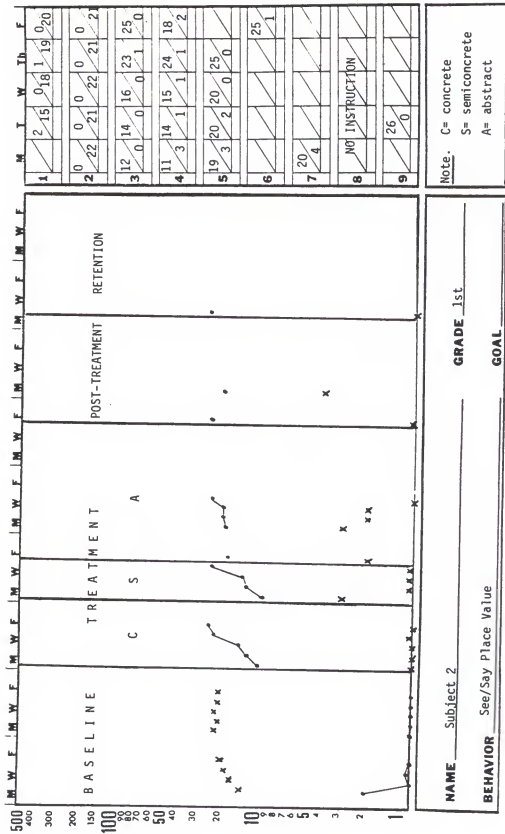


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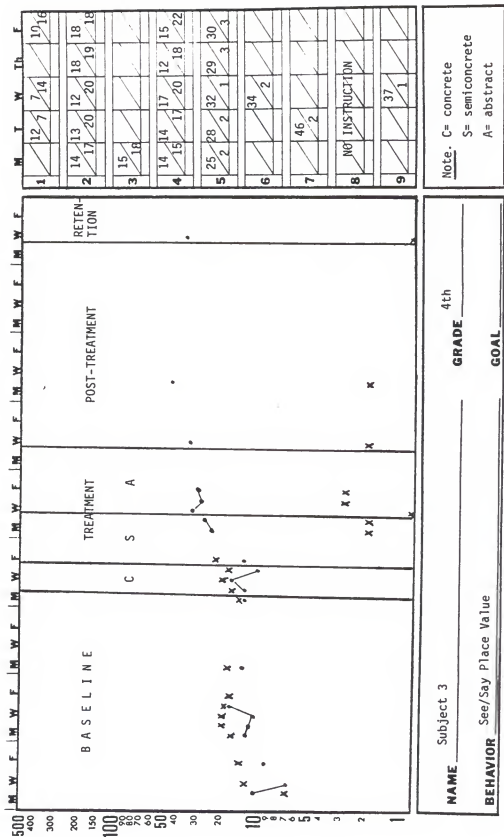


Figure 2--continued.

significant and immediate improvement during instruction in either concrete or semiconcrete instruction. For subjects 1 and 3, this occurred during semiconcrete instruction; for subject 2 this occurred during concrete instruction. The celerations for these students' performance were very good. Celerations for subjects 1, 2, and 3 were X2.35, X1.25, and X1.67, respectively. The daily measures and celerations obtained suggest that the treatment was effective for these three subjects. Additionally, the data demonstrate skill maintenance and retention. Since the posttreatment measures occurred in the students' home schools, generalization to a new setting was also accomplished.

Conclusions. Due to the successful results obtained in the pilot study, a decision was made to implement the concrete to abstract place value teaching procedure on a larger scale and compare it to an alternative teaching strategy. A group design was selected to facilitate the comparison and increase the generalizability of the findings.

Training Procedures (Phase Two)

Prior to beginning the group study, all teachers involved in implementation attended several training workshops. The teachers were introduced to the direct instruction model, effective teaching research, and the concrete to abstract teaching sequence. Moreover, they

were trained to implement the instructional activities. Training procedures are outlined in Appendix K. Teachers had to demonstrate mastery in administering the instructional procedures before beginning the actual treatment with study participants. An interval recording system was implemented to measure each teacher's ability to follow the script, follow the sequence, pace fluently, and use materials correctly. While each teacher demonstrated a lesson, two independent observers listened to a cassette tape that designated 20-second intervals with a beep. Each time the beep was heard, the observers recorded either appropriate or inappropriate teacher behaviors on the provided form (see Appendix L). Interobserver agreement was determined and is reported in Chapter IV.

Instructional Implementation (Phase Three)

An initial screening device was administered to determine which subjects were eligible to participate in the study. Then subjects were randomly assigned to the treatment and control groups and instruction began.

One-level group. The place value skill of identifying how many ones and/or tens are in a double digit number was taught to the control group using the four-step direct instruction model. Students received 15 minutes of direct instruction daily. A total of nine abstract level lessons were presented. Progression from one lesson to the next was dependent upon the student achieving a predetermined

criterion for skill performance. Instructional materials for the control group subjects included Scott, Foresman and Company; Macmillan Publishing Company; and teacher-made worksheets that present the skill symbolically without pictorial representations. The same teaching script for each lesson was used with all control group subjects. A precision teaching timing was administered each day immediately following the place value lesson.

Three-level group. The same place value skill of identifying how many ones and/or tens are located in a double digit number was taught to the experimental group using the four-step direct instructional model. Students received 15 minutes of direct instruction daily. Again, nine lessons were presented: three at the concrete level, three at the semiconcrete level, and three at the abstract level. Progression from one lesson to the next was dependent upon the student achieving a predetermined criterion for skill performance. Proportional aids for the three concrete level lessons included unifix plastic cubes, place value sticks, and teacher-made place value strips (Burton, 1984). Materials for the three semiconcrete level lessons were worksheets from Scott, Foresman and Company and Macmillan Publishing Company using pictorial representations of ones and tens in double digit numbers. The materials for abstract level lessons were the same worksheets used in three of the control group abstract

level lessons. The same teaching script for each lesson was used with all subjects in the three-level group. A precision teaching timing was administered each day immediately following the place value lesson.

Control variables. Several controls were built into the instructional component of the study. First, both the control and experimental group received 15 minutes of direct instruction daily and a 1-minute precision teaching timing on the same place value skill. Second, all students in the study had to achieve a predetermined criterion score on each lesson before progressing to the next. Third, teaching scripts were used to control for teacher effect (see Appendix M). Moreover, each teacher taught students from both the control and experimental group. Controlling these variables strengthens any significant findings resulting from the variation in treatment.

Posttreatment (Phase Four)

After instructional implementation was completed, a posttest was administered to each subject in both groups to measure skill acquisition. Additionally, each student's ability to generalize newly learned information to a different stimulus (i.e., three- and four-digit numbers) was tested. The following week skill acquisition and generalization were again measured. After 1 week without any instruction, practice, or maintenance activities, retention and generalization were tested.

Experimental Design and Analysis

The experimental design employed in this study is a 2X3 mixed design with one between (treatment) and one within (performance over time) group factor (Meyers, 1979). Subjects were randomly assigned to one of two groups each receiving a different level of the treatment variable. All subjects in the study received three levels (i.e., posttest, maintenance, and retention) of the performance over time variable (see Figure 3). The 2X3 mixed design combines the simplicity of the completely randomized design for the between group variable and the precision of the repeated measures design for the within group variable (Meyers, 1979).

A multivariate analysis of variance was computed to determine whether any significant differences exist among the levels of the experimental treatments. The two dependent measures, place value acquisition and place value generalization, were analyzed for each of the treatment combinations. Moreover, follow-up univariate analyses of variance were computed for skill acquisition and generalization. A .05 level of confidence was established for rejection of the null hypotheses.

		Performance Over Time		
		Posttest	Maintenance	Retention
Instructional Treatment	One-Level Instruction	$\bar{X}_{111}$	$\bar{X}_{112}$	$\bar{X}_{113}$
		$\bar{X}_{211}$	$\bar{X}_{212}$	$\bar{X}_{213}$
	Three-Level Instruction	$\bar{X}_{121}$	$\bar{X}_{122}$	$\bar{X}_{123}$
		$\bar{X}_{221}$	$\bar{X}_{222}$	$\bar{X}_{223}$

Note:

- Top $\bar{X}$ within each cell = acquisition probe.
 Bottom $\bar{X}$ within each cell = generalization probe.
 First subscript number = dependent variable
 (1 = acquisition, 2 = generalization).
 Second subscript number = instructional treatment
 (1 = one-level, 2 = three-level).
 Third subscript number = time measure (1 = posttest, 2 = maintenance, 3 = retention).

Figure 3
 Experimental Design: 2X3 Mixed Design With
 One Between and One Within Group Factor

CHAPTER IV RESULTS

The purpose of Chapter IV is to present data obtained to answer the experimental questions in this study. First, the interobserver agreement obtained from the teacher training procedures is reported. Second, statistical analyses of the data are discussed. Results from the multivariate analysis of variance (MANOVA) are provided. Third, the univariate analyses of variance results are discussed related to the null hypotheses reported in Chapter III. Related findings including generalization gain scores and error pattern analysis are presented in the remaining portion of the chapter.

Teacher Training Interobserver Agreement

As mentioned in Chapter III, an interval recording system was implemented to verify adequate teacher skill for delivering the place value lessons. Interval agreement (i.e., $[\text{Agreements} \div (\text{Agreements} + \text{Disagreements})] \times 100 = \text{Percent of Agreement}$) was calculated using the interval by interval or point by point method (Tawney & Gast, 1984). Miller's (1975) .90 standard was used as the criterion for adequate agreement. The overall interobserver agreement for the five teacher trainees in this study was .99. Thus,

the criterion was met. Individual scores for each training component (i.e., script, sequence, pacing, and materials) are presented in Table 5.

Table 5
Interobserver Agreement on Training Lessons

Trainee	Script	Sequence	Pacing	Materials	Total Percentage
Teacher 1	15/15	15/15	15/15	15/15	60/60 = 100%
Teacher 2	25/25	25/25	25/25	24/25	99/100 = 99%
Teacher 3	22/22	22/22	22/22	19/22	85/88 = 97%
Teacher 4	14/14	14/14	14/14	14/14	56/56 = 100%
Teacher 5	8/8	8/8	8/8	8/8	32/32 = 100%
Overall Interobserver Agreement					332/336 = 99%

Statistical Analysis of the Data

The means and standard deviations of the data from the two dependent measures (i.e., acquisition and generalization) are reported in Table 6 by treatment and performance over time. One-level instruction refers to the control group receiving abstract instruction, whereas three-level instruction refers to the experimental group receiving concrete, semiconcrete, and abstract instruction.

Table 6
Means and Standard Deviations for
Each Dependent Measure by Experimental Condition

Instructional Treatment	Dependent Variables	Performance Over Time		
		Posttest	Maintenance	Retention
One-Level Instruction	Acquisition	16.08 (7.329)	18.75 (1.712)	16.5 (5.214)
	Generalization	6.167 (4.629)	8.5 (6.186)	8.333 (6.651)
Three-Level Instruction	Acquisition	19.83 (.5774)	19.92 (.2887)	19.92 (.2887)
	Generalization	9.75 (8.081)	11 (6.382)	11.58 (5.807)

Overall Effect for Instructional Treatment

The MANOVA procedure was used to test the effects of the independent variables on both dependent variables simultaneously taking into consideration the correlation between acquisition and generalization. The MANOVA controls the experiment-wise Type I error rate by comparing the effects of the independent variables on a linear combination of the two dependent measures (O'Shea, 1984). In this analysis, the A between group factor was instructional treatment and the B within group factor was performance over time. Table 7 summarizes the MANOVA for one- and three-level instruction.

Table 7
Summary of Multivariate Analysis of Variance
for One- and Three-Level Instruction

Source	Hotelling's T^2	F	df	<u>p</u>
Between Subjects A	9.41713	4.49	2,21	.0237*
Within Subjects B	11.3374	2.45	4,19	.0816
Interaction AXB	4.76209	1.03	4,19	.4182

* Significant at the p<.05 level.

The MANOVA yielded a significant main effect for instructional treatment [$F(2, 21)=4.49$; p=.0237]. No instructional treatment by performance over time interaction effect was found [$F(4, 19)=1.03$; p=.4182]. Although the independent variable, instructional treatment, revealed a differential effect, the MANOVA does not provide information that delineates the nature of the effect with respect to the dependent measures. In other words, the MANOVA does not specify how the two groups differed on the acquisition and generalization measures. To obtain such information and thus accept or reject the null hypotheses reported in Chapter III, a follow-up procedure was applied to the data.

Examination of Dependent Measures

The univariate analysis of variance, an appropriate follow-up procedure to use with significant MANOVA results, was selected to examine the two dependent measures and thus accept or reject the four null hypotheses listed below.

1. There will be no significant difference between students receiving three- and one-level instruction in place value acquisition scores obtained immediately following treatment.
2. There will be no significant difference between subjects receiving three- and one-level instruction in place value acquisition scores on a maintenance probe.
3. There will be no significant difference between students receiving three- and one-level instruction in place value acquisition scores obtained 3 weeks after instruction.
4. There will be no significant difference between students receiving three- and one-level instruction in place value generalization scores obtained over time.

Two univariate analyses of variance were performed: one to determine acceptance or rejection of hypotheses 1 through 3 dealing with acquisition and one to determine acceptance or rejection of hypothesis 4 dealing with generalization (see Tables 8 and 9).

Table 8
Summary Table for the Univariate Analysis
of Variance for Acquisition

Source	df	SS	F	<u>p</u>
Between Subjects				
A	1	138.88	8.79	.0072*
Error	22	347.77		
Within Subjects				
B	2	25.75	.98	.3845
AXB	2	23.69	.90	.4143
Error	44	579.88		

* Significant at the $p < .05$ level.

Table 9
Summary Table for the Univariate Analysis
of Variance for Generalization

Source	df	SS	F	<u>p</u>
Between Subjects				
A	1	174.22	1.66	.2111
Error	22	2310.22		
Within Subjects				
B	2	58.02	3.45	.0406*
AXB	2	3.69	.22	.8038
Error	44	370.27		

* Significant at the $p < .05$ level.

A significant main effect for the instructional treatment variable A was found on the acquisition measures [$F(1, 22)=8.79$; $p=.0072$]. Therefore, hypotheses 1, 2, and 3 are rejected. Students who received the concrete to abstract instructional sequence identified ones and tens in double digit numbers significantly better than the students who received abstract level instruction.

No main effect difference for the instructional treatment variable A was found on the generalization measure [$F(1, 22)=1.66$; $p=.2111$]. Therefore, hypothesis 4 was retained. Students in the experimental and control groups were statistically similar when identifying ones and tens in three- and four-digit numbers. An increased within-group variability among the experimental subjects contributed to this finding.

Although not related to the study questions, it is interesting to note that a significant main effect for the performance over time variable B was found only on the generalization measure [$F(2, 44)=3.45$; $p=.0406$]. Fisher's least significant difference test (Keppel, 1982) was used to analyze the marginal means for the posttest, maintenance, and retention performances. Fisher's follow-up procedure specifies the location of the differences found on the generalization measure over time. The mean score for the posttest was significantly less than both the mean scores for the maintenance and retention measures.

Therefore, the subjects performed significantly better on the maintenance and retention generalization measures than they did on the generalization posttest measure. Table 10 displays these findings using Duncan's (1955) underlining notation.

Table 10
Results of Fisher's Least Significant Difference Test
for Performance Over Time Main Effects on
Acquisition and Generalization

Dependent Variable	Performance Over Time		
	Posttest	Maintenance	Retention
Acquisition	17.958	19.333	18.208
Generalization	7.9583	<u>9.7500</u>	<u>9.9583</u>

No interaction was found between performance over time and instructional treatment for either dependent measure ($[F(2, 44) = .90; p = .4143]$ and $[F(2, 44) = .22; p = .8038]$, respectively). The experimental group performed better than the control group at each level of the performance over time measure (i.e., posttest, maintenance, retention). The difference between groups at each of these levels was approximately the same. Figures 4 and 5 graphically illustrate acquisition and generalization cell means and reflect the lack of interaction.

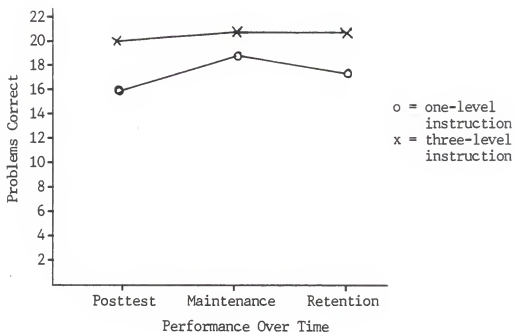


Figure 4
Mean Acquisition Scores Over Time

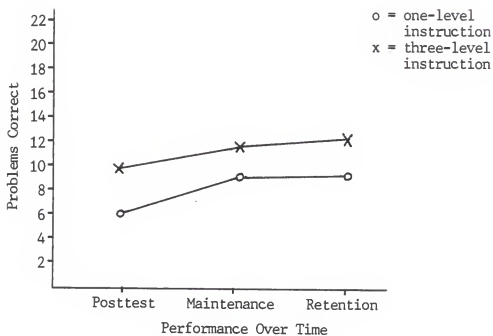


Figure 5
Mean Generalization Scores Over Time

Related Findings

Statistical analysis of the data was used to answer the identified research questions. Further analysis included the calculation of mean gain scores to identify possible trends in the generalization test scores. Additionally, an analysis of student error patterns on the generalization probes was performed. These results are summarized below.

Generalization Gain Scores

Gain scores from the generalization screening instrument to the generalization posttest were calculated separately for the experimental and control groups. Students in the experimental group who received the concrete, semiconcrete, and abstract instruction had a mean gain score of 47.08%. They completed an average of 9.42 more problems correct on the posttest than on the screening instrument. Students in the control group who received all abstract level instruction had a mean gain score of 26.25%. They completed an average of 5.25 more problems correct on the posttest than on the screening instrument.

Gain scores calculated from the screening instrument to the maintenance probe were 53.33% for the experimental group and 37.92% for the control group. The experimental group increased their number of correctly answered problems by a mean of 10.67. The control group performed an average

of 7.58 more problems correctly on the maintenance probe than they did on the initial screening.

Generalization gain scores were also calculated for both groups between the screening and retention measures. The experimental group had a mean gain score of 56.25% or 11.25 more problems correct. The control group had a mean gain score of 37.08% or 7.42 more problems correct (see Table 11).

Table 11
Mean Generalization Gain Scores

	Control Group	Experimental Group
Screening to Posttest	26.25% (5.25 problems)	47.08% (9.42 problems)
Screening to Maintenance	37.92% (7.58 problems)	53.33% (10.67 problems)
Screening to Retention	37.08% (7.42 problems)	56.25% (11.25 problems)

Both groups improved their generalization performance over time. More important, however, is the fact that on the posttest, maintenance, and retention gain score measures the students in the experimental group who received concrete to abstract instruction performed 15.41% to 20.83% better than the students in the control group.

Generalization Error Pattern Analysis

Since students in this study were not directly taught how to determine the number of tens and ones in three- and four-digit numbers, they had to develop strategies to perform this generalization task. A variety of error patterns emerged. These are noted in Table 12. The most common error pattern was identifying the ones correctly and incorrectly identifying the first number (i.e., the number on the far left) as the tens. For example, in the number 125 the student would say there was 1 ten and 5 ones. In the number 3,578, the student would say there were 3 tens and 8 ones. In this analysis, nine different subjects made this error with a total of 20 occurrences.

Summary

The teacher training phase of this study resulted in 99% interobserver agreement which surpassed the predetermined 90% criterion. Clearly, the teachers mastered the skills needed to deliver the place value lessons. The multivariate analysis of variance and two univariate analyses of variance were used to answer the research questions. The experimental group performed significantly better than the control group on acquisition of the place value skill immediately following the instructional unit (i.e., posttest), 1 week later (i.e., maintenance), and 3 weeks after instruction ceased (i.e.,

Table 12
Generalization Error Patterns

Subjects	looked at 1st 2 #s	1st # tens, last 2 ones	1st # ones, 2nd # tens	double digit answers	1st 2 #s tens, last 2 #s ones	1st 2 #s tens, copied whole #	1st 2 #s tens, copied #s ones correct
Control							
1			3				
2							
3							
4							
5	2						
6					1		
7							
8							
9				3			
10							3
11			3				
12							
Treatment							
13							
14							
15							
16						1	
17					1		
18							
19		3			1		
20							
21							3
Total occurrences	5	3	3	3	2	1	6

Table 12--continued.

Subjects	ones correct, tens random	tens correct, ones random	ones correct, tens random combination of 2 #'s	ones correct, tens 1st #	reversed ones and tens	random guessing
Control						
1						
2				3		
3				1		2
4			3			
5						
6		1				
7				1		1
8						
9						
10	1				1	
11						
12						1
Treatment						
13						
14	1					
15				3		
16				2		
17				3		2
18				2		
19				2		
20						
21				3		
Total occurrences	2	1	3	20	1	6

Note: Students who scored less than 70% on the generalization measure were included in the error pattern analysis.

retention). No statistically significant difference was found between the two groups on the generalization probes.

Generalization gain scores calculated to reflect growth between the screening and posttest measures for the experimental and control groups were 47.08% and 26.25%, respectively. The gain scores reflecting change between the screening and maintenance devices were 53.33% for the experimental group and 37.92% for the control group. The gain scores between the screening and retention generalization measures were 56.25% for the experimental group and 37.08% for the control group. Thus, the experimental group performed 20.83%, 15.41%, and 19.17% better than the control group on the generalization task.

Student error patterns emerged on the generalization measures. The most frequent error, with 20 occurrences, was identifying the ones correctly in three- and four-digit numbers and misidentifying the tens.

These combined results suggest the concrete to abstract teaching sequence was more effective for place value acquisition than the abstract level teaching. Additionally, a trend evolved indicating the experimental group performed better on generalization tasks than the control group.

CHAPTER V DISCUSSION

Results reported in the previous chapter have implications for teaching mathematics to learning disabled students. The purpose of Chapter V is to discuss these implications and offer plausible explanations for their existence. Theoretical considerations related to earlier research are presented first. Then, findings and practical implications obtained in each phase of the study are discussed and summarized. Finally, suggestions for future research are offered.

Theoretical Considerations

To date, the theory suggesting that mathematical instruction should begin with concrete learning experiences and then progress to semiconcrete and finally abstract levels of understanding lacks empirical support. Most research related to this topic has compared one-level instruction to another (i.e., concrete vs. abstract or semiconcrete vs. abstract or concrete vs. semiconcrete). Additionally, previous work primarily focuses on the performance of regular education students and excludes the learning disabled population.

St. Martin's (1975) research provides some empirical data regarding the concrete to abstract sequence theory. His study specifically addresses the sequencing of the three levels of instruction. The population included regular education students; thus his results have limited implications to exceptional students.

The current study provides needed data to further examine the accuracy of the concrete to abstract teaching theory. Furthermore, the study specifically limits the population to learning disabled students. The foundation and need for further exploration have been set.

Discussion of the Findings and Practical Implications

The purpose of this study was to compare the acquisition of an initial place value skill when presented in a concrete, semiconcrete, abstract teaching sequence to acquisition of the same skill when presented abstractly. Time and generalization measures were also included.

Four phases were developed to establish experimental control, ensure research reliability and validity, and thus achieve the stated purpose with a high level of confidence in the results. The findings obtained during each phase are discussed in terms of their practical implications.

Phase One (Pilot Study)

Phase one consisted of a pilot study designed to document the possible effectiveness of teaching an initial

place value skill (i.e., identifying ones and tens in double digit numbers) to learning disabled students using the concrete to abstract teaching sequence. Skill acquisition, retention, and generalization to a new setting were examined. The findings demonstrate that the concrete to abstract sequence was effective for skill acquisition. The subjects' percentage gain scores on a criterion referenced measure were 80%, 90%, and 40%. These increases represent the difference between receiving an F and a B on the place value test for two students and an F and an A for one student (i.e., assuming 40% is an F, 80% is a B, and 90% is an A).

Retention scores, obtained 3 weeks after instruction in a different classroom setting, further substantiate the benefit of the concrete to abstract intervention. All three students earned 80% on the retention measure. Closer analysis of the retention errors revealed an interesting phenomenon. All three students missed the same two questions. Both questions asked how many tens there were in single digit numbers (i.e., 6 and 4). The instructional lessons presented to the students only provided practice with double digit numbers. The discrepancy between what was taught and what was measured accounts for the problems missed on the posttest and retention measures. Thus, one can assume the few difficulties demonstrated were a result

of the testing instrument rather than lack of skill acquisition or retention.

In addition to the criterion-reference measures, daily 1-minute timings provided data for analysis. Learning disabilities teachers must continually concern themselves with the rate of student learning. Teaching efficiency and how quickly skill fluency is obtained becomes very important when teaching students who are typically 2 to 3 years behind their peers. In this pilot study, the students' celerations (i.e., rate of learning) ranged from X1.25 to X2.35. These rates meet the generally accepted X1.25 criterion (White & Haring, 1980). The student rates were maintained after instruction ceased.

The daily 1-minute timings revealed another interesting learning pattern. Each student made a significant gain at one particular point in the instructional sequence. One student demonstrated this gain during concrete instruction, while the other two students demonstrated similar gains during semiconcrete instruction. Thus, the transition to understanding an abstract level probe based on concrete or semiconcrete instruction seemed to occur suddenly and conclusively, but at varying points within the concrete to abstract sequence. Once a student realizes the abstract representations (i.e., numbers) are directly related to the manipulatives and/or pictures presented during the lessons,

understanding is achieved immediately and maintained. This finding supports the recommendation (Underhill et al., 1980) to pair manipulatives with the symbolic processes that describe them to promote conceptual understanding.

Phase one findings suggest that students can be taught to identify and understand the concept of ones and tens in double digit numbers using the concrete to abstract teaching sequence in a short amount of time. Only 15 minutes per day for 9 to 14 days were required for all three students to reach criterion on the scripted lessons. The findings also suggest that the instructional sequence promotes skill retention, a frequent problem among learning disabled students, and skill generalization to a new setting. The latter finding is particularly relevant to learning disabled students who are mainstreamed to regular classroom settings.

Phase Two (Teacher Training)

The pilot study's success resulted in a decision to compare the concrete to abstract teaching sequence to the traditional abstract approach using a larger population. To obtain enough learning disabled students for a group design, five teachers were asked to participate in the study. The involvement of this many teachers necessitated systematic training to ensure that subjects attending different classes received the same treatment.

The training sessions for all participating teachers began with a lecture format to introduce effective teaching research, direct instruction, and the concrete to abstract mathematical sequence. The training sessions also included a demonstration lesson. Thus, the teachers observed proper use of the script and manipulatives. The last part of the teacher training was perhaps the most important. Each teacher taught a sample lesson using mock students. Independent observers measured the teachers' ability to follow the script, follow the sequence, use fluent pacing, and correctly manipulate the materials.

These training sessions were very successful. In addition to the 99% interobserver agreement, the teachers expressed confidence in their abilities to implement the lessons. Fortunately, four out of the five teachers had taught scripted lessons (i.e., Distar) previous to this project. Familiarity with this format was probably helpful. Teachers unfamiliar with scripted lessons may need to practice several lessons before feeling confident.

Approximately 4 to 5 hours of formal training time were adequate preparation for the teachers. It should be noted, however, that in addition to the formal training each teacher reviewed the instructional materials and practiced the script on her own time. Additionally, the teachers who participated in this study were recommended due to cooperative attitudes and excellent teaching

skills. Undoubtedly, the systematic training coupled with skilled and positive teachers enhanced the quality of this research.

Phase Three (Implementation)

Phase three of the study consisted of implementing nine place value lessons to the experimental group and nine place value lessons to the control group. Each lesson took 15 minutes and incorporated advance organizers, demonstration/modeling, guided practice, and independent practice. The students had to achieve a predetermined criterion before advancing to the next lesson. A small group format (i.e., two to four students with one teacher) was used to present the lessons. Informal feedback from the participating teachers indicated that management of the manipulatives would be difficult with groups containing more than four students. Exceptions to the small group delivery system occurred if a student was absent or failed to meet the lesson's criterion. In such cases, one-to-one instruction was provided so the student could rejoin the small group for the subsequent lesson.

The implementation procedures established for this study provide tight experimental control frequently ignored in applied research. The studies reviewed in Chapter II (Armstrong, 1972; Fennema, 1972; Prigge, 1978; Scott & Neufeld, 1976; Smith et al., 1980) failed to adequately control for teacher effect. Students in one class were

compared to students in other classes who had different teachers. Such comparisons lack control for many confounding variables (e.g., teacher-student interpersonal communication, teaching style). Armstrong (1972) used slides and tapes to present the information to be learned in an attempt to control for teacher effect. Student behavior problems, however, prevented the continuation of this presentation format. Teachers began providing the instruction to increase the likelihood of appropriate classroom behavior. Clearly, the failure to control for differences among teachers (e.g., amount of rapport with students, teaching ability, behavior management skills) reduces research credibility.

Several tactics were included in the implementation of this research to control for teacher differences. First, all materials needed for the lessons were provided to the teachers. Thus, identical manipulatives and worksheets were used with all students. Second, to control for content delivery and teacher language all 18 lessons were scripted. Additionally, the lessons for both groups provided direct instruction and verbal praise. These two strategies have been very successful with learning disabled students (Lovitt, 1977; Madsen, Becker, & Thomas, 1968). Third, time restrictions were placed on the lessons to ensure the same amount of practice for each subject. Fourth, each teacher was responsible for teaching both an

experimental and control group. This balancing increased confidence that the treatment was responsible for differing performance between the two groups rather than variations in teacher style.

The systematic planning and implementation resulted in valuable data collection. The researcher hopes future explorations will replicate the procedural control achieved in this study.

Phase Four (Posttreatment)

Posttreatment measures were gathered immediately following instruction, 1 week after instruction, and 3 weeks after instruction. At these three points in time the students were tested individually on acquisition of the skill taught and generalization to an untaught, higher level skill. No additional place value review or instruction occurred during the posttreatment phase.

Acquisition findings. As reported in Chapter IV, the students who received the concrete to abstract instruction acquired the place value skill significantly ($p < .05$) better than the students who received traditional abstract instruction. The significantly better performance occurred on all three measures (i.e., posttest, maintenance, and retention). The inclusion of manipulatives (i.e., plastic cubes, popsicle sticks, and place value strips) and pictures increased the students understanding and their ability to identify ones or tens in double digit numbers.

The teachers informally reported that the experimental group students really seemed to enjoy using the manipulatives. Motivation, however, was observed to be high among students in both groups. The quick pace, immediate feedback, and built-in verbal praise were specifically included in all lessons to promote student interest and increase on-task behavior.

Generalization findings. The experimental group performed better on the generalization posttest, maintenance, and retention measures than the control group. The within-group variability, however, prevented this difference from being statistically significant. Although some students did generalize with 100% accuracy; others did not generalize at all. Those who failed to generalize substantiate earlier research (Stokes & Baer, 1977) which suggests that special education students must be taught to generalize. The difference in gain scores reported in Chapter IV denotes a trend toward spontaneous generalization among the experimental group students. Thus, instructionally it makes sense to use the concrete to abstract sequence when generalization is part of the teaching objective. Teachers must, however, be prepared to incorporate additional generalization practice for some students.

Conclusions. The concrete to abstract teaching sequence is more effective than abstract teaching with

learning disabled students who are acquiring initial place value skills. Moreover, the data suggest skill retention can be achieved for at least 3 weeks. This is particularly appealing when one considers the sequential nature of mathematics and the importance of mastering prerequisite skills. The effectiveness of the concrete to abstract sequence for promoting generalization is still inconclusive. Further research should be done in this area.

Research Questions

The research questions listed below were successfully answered in this study.

1. Is the concrete to abstract instructional sequence more effective than the abstract only approach in teaching the acquisition of place value skills?
2. Is the concrete to abstract instructional sequence more effective than the abstract only approach in maintenance of the place value skill?
3. Is the concrete to abstract instructional sequence more effective than the abstract only approach for promoting retention of the place value skill?
4. Is the concrete to abstract instructional sequence more effective than the abstract only approach in promoting generalization to higher level place value skills?

The data show that questions one through three can be

answered affirmatively. Question four cannot be answered affirmatively from a statistical point of view. Analysis of the raw data, however, suggests the concrete to abstract sequence was more effective than the abstract only approach for promoting generalization.

Summary of Practical Implications

Numerous practical implications for mathematics teachers and researchers were provided in the preceding discussion. The most salient implications from the four study phases are listed below.

1. The concrete to abstract sequence promotes skill acquisition.
2. The concrete to abstract sequence promotes skill retention.
3. The concrete to abstract sequence promotes generalization to another setting.
4. The concrete to abstract sequence produces a high learning rate which increases teaching efficiency.
5. Generalization to higher level skills must be directly taught to some students.
6. The concrete to abstract drill results in higher test scores than abstract level drill.
7. Once the association between concrete instruction and abstract understanding occurs, the student maintains the concept over time.

8. Nine lessons (i.e., three concrete, three semiconcrete, and three abstract) are sufficient for obtaining skill acquisition, maintenance, and retention.
9. Teacher training and scripted lessons provide experimental control when several teachers are involved in research implementation.

In summary, mathematics instructors teaching initial place value skills should use the concrete to abstract sequence. A direct instruction format is effective for systematically presenting the sequence during small group instruction (i.e., two to four students per group). Basal mathematics programs should be supplemented with concrete and semiconcrete activities to facilitate acquisition, maintenance, and retention. Instructors can expect increased student achievement when the concrete to abstract teaching intervention is used.

These implications are appropriate for teaching place value to learning disabled students in elementary and middle school grades. Caution should be used before generalizing the results and their implications to other populations or other mathematical skills.

Future Research

The findings of this study demonstrate the need for continued research involving the concrete to abstract

mathematical intervention. Suggestions for future research include further testing of higher level place value skills, applying the concrete to abstract teaching sequence to other mathematical skills, expanding the population to include other exceptionalities, and additional exploration of generalization factors.

Higher Level Place Value Skills

The present study examined the initial place value skill of identifying tens and ones in double digit numbers using both single subject and group designs. Replication of these procedures with higher level place value skills would add to the existing data-base. An initial follow-up study might expand the instruction to include three-digit numbers containing hundreds, tens, and ones. Slight modifications in the teaching script and material adaptations would facilitate this programmatic research. Additional concrete to abstract studies could be designed to test mathematical skills related to place value such as addition with carrying and subtraction with borrowing. It would also be interesting to explore the teaching sequence's effect on hierarchical learning. For example, do students who receive the concrete to abstract teaching for initial place value skill training acquire higher level place value skills more quickly than students who are taught abstractly? Does the concrete to abstract approach

have a long term carry-over effect? Longitudinal studies may provide answers to these questions.

Other Mathematical Skills

In addition to further place value study, the concrete to abstract sequence should be tested with other mathematical skills. To date, two replications of the place value pilot study have occurred to test the three-level sequence with addition skills. Robinson (1986) reports excellent findings using the concrete to abstract sequence to teach addition facts with sums to 18. Kelly (1986) also found the procedure successful for teaching money sums.

Numerous skills remain untested. Among these are readiness skills (e.g., counting and one-to-one correspondence), subtraction, multiplication, division, fractions, and measurement. Single subject and/or group research would be beneficial for determining whether the concrete to abstract teaching sequence is effective across skills or whether it is more applicable to some areas than others.

Population Expansion

Another important question to address with future research is whether the concrete to abstract teaching intervention is effective with a variety of special education populations. In this study, its effectiveness with learning disabled students has been documented. Would

similar results be obtained with educably mentally handicapped and emotionally handicapped students? Would the effect be different for physically involved students who observed the manipulation of objects rather than actually participating in their movement?

Another experimental variable related to the research population is age. Exploration regarding the intervention's appropriateness with high school-aged students is needed. Suydam states that "achievement is enhanced across a variety of topics, at every grade level K-8, at every achievement level, at every ability level" (1984, p. 27) when manipulative materials are used for mathematics instruction. She does not, however, specifically address high school students using the concrete to abstract sequence. Future research could test the effectiveness of this intervention with handicapped secondary students who are working on lower level mathematical skills and also those who are enrolled in higher level classes such as algebra or geometry. Traditionally, special education researchers have emphasized intervention studies with elementary-aged students. The researcher hopes future research will provide equitable time to secondary-aged students.

Generalization Research

The generalization results in this study clearly indicate a need for further research. Generalization to a

new setting was successful in the pilot study, while generalization to an untaught, higher level skill did not reach significance in the group design study. These findings raise questions that need to be examined further. For example, which students need to be taught generalization skills? How much time should be spent on such teaching? If the concrete to abstract intervention was modified, would generalization be more likely to occur across environments and skills? Does the concrete to abstract sequence promote generalization in some skills and not in others? These and other related questions are very important to special educators and their students. A systematic approach to programmatic research is needed to draw conclusions beyond the scope of this study.

APPENDIX A
TEACHER-MADE SCREENING DEVICE

Part One

- | | | |
|--------|-----------------------------------|-------|
| 1. 23 | How many tens are in this number? | _____ |
| 2. 45 | How many tens are in this number? | _____ |
| 3. 54 | How many ones are in this number? | _____ |
| 4. 28 | How many ones are in this number? | _____ |
| 5. 31 | How many tens are in this number? | _____ |
| 6. 17 | How many ones are in this number? | _____ |
| 7. 78 | How many ones are in this number? | _____ |
| 8. 82 | How many ones are in this number? | _____ |
| 9. 32 | How many tens are in this number? | _____ |
| 10. 18 | How many ones are in this number? | _____ |
| 11. 20 | How many tens are in this number? | _____ |
| 12. 19 | How many tens are in this number? | _____ |
| 13. 24 | How many tens are in this number? | _____ |
| 14. 33 | How many ones are in this number? | _____ |
| 15. 76 | How many tens are in this number? | _____ |
| 16. 81 | How many tens are in this number? | _____ |
| 17. 12 | How many ones are in this number? | _____ |
| 18. 10 | How many ones are in this number? | _____ |
| 19. 67 | How many tens are in this number? | _____ |
| 20. 49 | How many tens are in this number? | _____ |

Part Two

- | | | | |
|-----|------|-----------------------------------|-------|
| 1. | 323 | How many tens are in this number? | _____ |
| 2. | 4645 | How many tens are in this number? | _____ |
| 3. | 154 | How many ones are in this number? | _____ |
| 4. | 628 | How many ones are in this number? | _____ |
| 5. | 3531 | How many tens are in this number? | _____ |
| 6. | 8717 | How many ones are in this number? | _____ |
| 7. | 178 | How many ones are in this number? | _____ |
| 8. | 482 | How many ones are in this number? | _____ |
| 9. | 332 | How many tens are in this number? | _____ |
| 10. | 7818 | How many ones are in this number? | _____ |
| 11. | 220 | How many tens are in this number? | _____ |
| 12. | 319 | How many tens are in this number? | _____ |
| 13. | 6124 | How many tens are in this number? | _____ |
| 14. | 933 | How many ones are in this number? | _____ |
| 15. | 1476 | How many tens are in this number? | _____ |
| 16. | 6281 | How many tens are in this number? | _____ |
| 17. | 112 | How many ones are in this number? | _____ |
| 18. | 3310 | How many ones are in this number? | _____ |
| 19. | 8767 | How many tens are in this number? | _____ |
| 20. | 549 | How many tens are in this number? | _____ |

APPENDIX B
TEACHER-MADE POSTTEST FORM A

Part One

- | | | |
|--------|-----------------------------------|-------|
| 1. 43 | How many ones are in this number? | _____ |
| 2. 54 | How many tens are in this number? | _____ |
| 3. 51 | How many tens are in this number? | _____ |
| 4. 12 | How many tens are in this number? | _____ |
| 5. 23 | How many ones are in this number? | _____ |
| 6. 42 | How many ones are in this number? | _____ |
| 7. 17 | How many ones are in this number? | _____ |
| 8. 58 | How many tens are in this number? | _____ |
| 9. 39 | How many ones are in this number? | _____ |
| 10. 94 | How many tens are in this number? | _____ |
| 11. 19 | How many tens are in this number? | _____ |
| 12. 30 | How many ones are in this number? | _____ |
| 13. 10 | How many tens are in this number? | _____ |
| 14. 56 | How many ones are in this number? | _____ |
| 15. 84 | How many ones are in this number? | _____ |
| 16. 61 | How many tens are in this number? | _____ |
| 17. 16 | How many ones are in this number? | _____ |
| 18. 98 | How many tens are in this number? | _____ |
| 19. 28 | How many tens are in this number? | _____ |
| 20. 46 | How many ones are in this number? | _____ |

Part Two

- | | | | |
|-----|------|-----------------------------------|-------|
| 1. | 324 | How many ones are in this number? | _____ |
| 2. | 4645 | How many tens are in this number? | _____ |
| 3. | 157 | How many tens are in this number? | _____ |
| 4. | 528 | How many ones are in this number? | _____ |
| 5. | 3567 | How many tens are in this number? | _____ |
| 6. | 8237 | How many tens are in this number? | _____ |
| 7. | 188 | How many ones are in this number? | _____ |
| 8. | 341 | How many ones are in this number? | _____ |
| 9. | 876 | How many tens are in this number? | _____ |
| 10. | 7345 | How many ones are in this number? | _____ |
| 11. | 276 | How many tens are in this number? | _____ |
| 12. | 332 | How many tens are in this number? | _____ |
| 13. | 6124 | How many tens are in this number? | _____ |
| 14. | 933 | How many ones are in this number? | _____ |
| 15. | 1476 | How many ones are in this number? | _____ |
| 16. | 6241 | How many tens are in this number? | _____ |
| 17. | 134 | How many ones are in this number? | _____ |
| 18. | 3310 | How many ones are in this number? | _____ |
| 19. | 8867 | How many ones are in this number? | _____ |
| 20. | 534 | How many tens are in this number? | _____ |

APPENDIX C
TEACHER-MADE POSTTEST FORM B

Part One

- | | | |
|--------|-----------------------------------|-------|
| 1. 23 | How many ones are in this number? | _____ |
| 2. 43 | How many ones are in this number? | _____ |
| 3. 16 | How many tens are in this number? | _____ |
| 4. 58 | How many ones are in this number? | _____ |
| 5. 33 | How many ones are in this number? | _____ |
| 6. 47 | How many ones are in this number? | _____ |
| 7. 85 | How many tens are in this number? | _____ |
| 8. 74 | How many tens are in this number? | _____ |
| 9. 10 | How many ones are in this number? | _____ |
| 10. 73 | How many tens are in this number? | _____ |
| 11. 50 | How many ones are in this number? | _____ |
| 12. 82 | How many ones are in this number? | _____ |
| 13. 51 | How many ones are in this number? | _____ |
| 14. 91 | How many tens are in this number? | _____ |
| 15. 37 | How many tens are in this number? | _____ |
| 16. 48 | How many ones are in this number? | _____ |
| 17. 85 | How many tens are in this number? | _____ |
| 18. 40 | How many ones are in this number? | _____ |
| 19. 37 | How many ones are in this number? | _____ |
| 20. 29 | How many tens are in this number? | _____ |

Part Two

- | | | | |
|-----|------|-----------------------------------|-------|
| 1. | 454 | How many ones are in this number? | _____ |
| 2. | 4755 | How many tens are in this number? | _____ |
| 3. | 154 | How many tens are in this number? | _____ |
| 4. | 568 | How many ones are in this number? | _____ |
| 5. | 7867 | How many tens are in this number? | _____ |
| 6. | 8637 | How many tens are in this number? | _____ |
| 7. | 438 | How many ones are in this number? | _____ |
| 8. | 871 | How many tens are in this number? | _____ |
| 9. | 986 | How many ones are in this number? | _____ |
| 10. | 2345 | How many ones are in this number? | _____ |
| 11. | 864 | How many ones are in this number? | _____ |
| 12. | 561 | How many tens are in this number? | _____ |
| 13. | 6743 | How many tens are in this number? | _____ |
| 14. | 763 | How many ones are in this number? | _____ |
| 15. | 3456 | How many ones are in this number? | _____ |
| 16. | 6873 | How many tens are in this number? | _____ |
| 17. | 154 | How many ones are in this number? | _____ |
| 18. | 3910 | How many tens are in this number? | _____ |
| 19. | 5467 | How many tens are in this number? | _____ |
| 20. | 875 | How many ones are in this number? | _____ |

APPENDIX D
TEACHER-MADE POSTTEST FORM C

Part One

- | | | |
|--------|-----------------------------------|-------|
| 1. 34 | How many ones are in this number? | _____ |
| 2. 45 | How many tens are in this number? | _____ |
| 3. 54 | How many tens are in this number? | _____ |
| 4. 53 | How many ones are in this number? | _____ |
| 5. 79 | How many ones are in this number? | _____ |
| 6. 90 | How many ones are in this number? | _____ |
| 7. 27 | How many tens are in this number? | _____ |
| 8. 31 | How many ones are in this number? | _____ |
| 9. 49 | How many tens are in this number? | _____ |
| 10. 17 | How many ones are in this number? | _____ |
| 11. 19 | How many ones are in this number? | _____ |
| 12. 93 | How many ones are in this number? | _____ |
| 13. 67 | How many tens are in this number? | _____ |
| 14. 10 | How many ones are in this number? | _____ |
| 15. 39 | How many tens are in this number? | _____ |
| 16. 68 | How many tens are in this number? | _____ |
| 17. 47 | How many ones are in this number? | _____ |
| 18. 26 | How many ones are in this number? | _____ |
| 19. 15 | How many ones are in this number? | _____ |
| 20. 40 | How many tens are in this number? | _____ |

Part Two

- | | | | |
|-----|------|-----------------------------------|-------|
| 1. | 453 | How many ones are in this number? | _____ |
| 2. | 4785 | How many tens are in this number? | _____ |
| 3. | 434 | How many tens are in this number? | _____ |
| 4. | 587 | How many ones are in this number? | _____ |
| 5. | 2347 | How many ones are in this number? | _____ |
| 6. | 6537 | How many tens are in this number? | _____ |
| 7. | 483 | How many ones are in this number? | _____ |
| 8. | 641 | How many tens are in this number? | _____ |
| 9. | 746 | How many ones are in this number? | _____ |
| 10. | 2235 | How many tens are in this number? | _____ |
| 11. | 874 | How many ones are in this number? | _____ |
| 12. | 461 | How many tens are in this number? | _____ |
| 13. | 6323 | How many tens are in this number? | _____ |
| 14. | 752 | How many tens are in this number? | _____ |
| 15. | 3456 | How many ones are in this number? | _____ |
| 16. | 6865 | How many tens are in this number? | _____ |
| 17. | 394 | How many ones are in this number? | _____ |
| 18. | 3980 | How many tens are in this number? | _____ |
| 19. | 4635 | How many ones are in this number? | _____ |
| 20. | 945 | How many ones are in this number? | _____ |

APPENDIX E
SAMPLE TEACHER-MADE WORKSHEET
FOR ABSTRACT INSTRUCTION

Directions: Fill in the place value charts.

1. 23

tens	ones

2. 43

tens	ones

3. 59

tens	ones

4. 15

tens	ones

5. 73

tens	ones

APPENDIX F
SEE/SAY PLACE VALUE PROBE

<u>21</u>	<u>42</u>	<u>39</u>	<u>46</u>	<u>19</u>	<u>27</u>	<u>94</u>	<u>63</u>
<u>43</u>	<u>76</u>	<u>56</u>	<u>21</u>	<u>98</u>	<u>34</u>	<u>65</u>	<u>78</u>
<u>32</u>	<u>16</u>	<u>16</u>	<u>45</u>	<u>56</u>	<u>87</u>	<u>34</u>	<u>26</u>
<u>78</u>	<u>32</u>	<u>22</u>	<u>87</u>	<u>56</u>	<u>12</u>	<u>65</u>	<u>74</u>
<u>23</u>	<u>56</u>	<u>87</u>	<u>54</u>	<u>90</u>	<u>43</u>	<u>56</u>	<u>75</u>
<u>21</u>	<u>56</u>	<u>98</u>	<u>45</u>	<u>76</u>	<u>23</u>	<u>98</u>	<u>67</u>
<u>23</u>	<u>14</u>	<u>16</u>	<u>87</u>	<u>65</u>	<u>45</u>	<u>75</u>	<u>34</u>
<u>34</u>	<u>65</u>	<u>23</u>	<u>57</u>	<u>87</u>	<u>34</u>	<u>32</u>	<u>12</u>
<u>35</u>	<u>87</u>	<u>56</u>	<u>47</u>	<u>26</u>	<u>85</u>	<u>35</u>	<u>67</u>
<u>34</u>	<u>65</u>	<u>23</u>	<u>67</u>	<u>34</u>	<u>12</u>	<u>13</u>	<u>68</u>
<u>29</u>	<u>62</u>	<u>97</u>	<u>24</u>	<u>10</u>	<u>29</u>	<u>38</u>	<u>43</u>
<u>24</u>	<u>78</u>	<u>54</u>	<u>23</u>	<u>98</u>	<u>43</u>	<u>24</u>	<u>76</u>

APPENDIX G
LEARNING CHARACTERISTICS OF PILOT STUDY SUBJECTS

Subject 1

Subject 1 was a second grade, learning disabled student. He had been retained twice; once in kindergarten and again in first grade. His cumulative mathematics average for the previous school year was a "D." A standardized test measure using the Kaufman Test of Educational Achievement (K-TEA) revealed below average performance in mathematics (i.e., standard score of 76; 5th percentile; 2nd stanine).

Subject 2

Subject 2 was a first grade, learning disabled student. His most recent mathematics report card grade was a "D" for "developmental" indicating a need for basic math instruction. The K-TEA revealed math performance only slightly below average (i.e., standard score of 99; 47th percentile; 5th stanine).

Subject 3

Subject 3 was a fourth grade, learning disabled student. He had been retained once in the second grade. His most recent mathematics report card grade was an "N"

for "needs improvement." The K-TEA revealed below average performance in mathematics (i.e., standard score of 73; 4th percentile; 2nd stanine).

Summary

In summary, all three students were elementary school-aged males experiencing varying degrees of difficulty with mathematics skills. Report card grades and scores obtained from the K-TEA revealed less than optimum performance.

APPENDIX H
PLACE VALUE PRETEST

1. 43 How many ones are in this number? _____
2. 37 How many ones are in this number? _____
3. 28 How many tens are in this number? _____
4. 13 How many tens are in this number? _____
5. 50 How many ones are in this number? _____
6. 18 How many ones are in this number? _____
7. 10 How many tens are in this number? _____
8. 7 How many tens are in this number? _____
9. 49 How many ones are in this number? _____
10. 2 How many tens are in this number? _____

APPENDIX I
PILOT STUDY PRECISION TEACHING PROBE

<u>41</u>	<u>26</u>	<u>64</u>	<u>31</u>	<u>61</u>	<u>85</u>	<u>27</u>	<u>11</u>	<u>16</u>	<u>67</u>	<u>48</u>
<u>23</u>	<u>52</u>	<u>58</u>	<u>43</u>	<u>71</u>	<u>90</u>	<u>66</u>	<u>48</u>	<u>87</u>	<u>32</u>	<u>15</u>
<u>19</u>	<u>15</u>	<u>82</u>	<u>39</u>	<u>17</u>	<u>64</u>	<u>65</u>	<u>37</u>	<u>92</u>	<u>16</u>	<u>84</u>
<u>43</u>	<u>46</u>	<u>12</u>	<u>93</u>	<u>81</u>	<u>57</u>	<u>68</u>	<u>99</u>	<u>92</u>	<u>17</u>	<u>18</u>
<u>43</u>	<u>46</u>	<u>12</u>	<u>93</u>	<u>81</u>	<u>18</u>	<u>91</u>	<u>68</u>	<u>13</u>	<u>47</u>	<u>71</u>
<u>29</u>	<u>56</u>	<u>28</u>	<u>17</u>	<u>69</u>	<u>14</u>	<u>24</u>	<u>30</u>	<u>50</u>	<u>31</u>	<u>82</u>
<u>41</u>	<u>84</u>	<u>55</u>	<u>92</u>	<u>27</u>	<u>43</u>	<u>18</u>	<u>65</u>	<u>79</u>	<u>26</u>	<u>31</u>
<u>68</u>	<u>45</u>	<u>17</u>	<u>91</u>	<u>33</u>	<u>54</u>	<u>98</u>	<u>47</u>	<u>51</u>	<u>83</u>	<u>92</u>
<u>16</u>	<u>10</u>	<u>24</u>	<u>34</u>	<u>81</u>	<u>10</u>	<u>29</u>	<u>75</u>	<u>65</u>	<u>37</u>	<u>70</u>

APPENDIX J
RETENTION PROBE

- | | | | |
|-----|----|-----------------------------------|-------|
| 1. | 53 | How many ones are in this number? | _____ |
| 2. | 38 | How many ones are in this number? | _____ |
| 3. | 18 | How many tens are in this number? | _____ |
| 4. | 45 | How many tens are in this number? | _____ |
| 5. | 40 | How many ones are in this number? | _____ |
| 6. | 48 | How many ones are in this number? | _____ |
| 7. | 10 | How many tens are in this number? | _____ |
| 8. | 6 | How many tens are in this number? | _____ |
| 9. | 35 | How many ones are in this number? | _____ |
| 10. | 4 | How many tens are in this number? | _____ |

APPENDIX K
TRAINING PROCEDURES

- I. Provide Advance Organizer
 - A. Training goals
 - B. Training expectations for progress
 - C. Training format
- II. Introduce Effective Teaching Research
- III. Introduce Direct Instruction Model
 - A. Identify four steps
 - 1. Provide advance organizer
 - 2. Model the skill
 - 3. Provide practice with feedback
 - 4. Provide independent practice
 - B. Identify characteristics of direct instruction
 - 1. Focus on academics
 - 2. Monitor student performance
 - 3. Provide opportunity for many correct responses
 - 4. Provide immediate feedback
 - 5. Maintain brisk pace
- IV. Introduce Concrete, Semiconcrete, and Abstract Modes of Instruction
 - A. Present examples
 - B. Present nonexamples
- V. Demonstrate Lesson One of Three-Level Treatment
- VI. Observe and Evaluate Performance of Trainees Using Interval Recording Format

APPENDIX L
INTERVAL RECORDING FORM

Teacher _____ Observer _____

1. Following Script
2. Following Sequence
3. Fluent Pacing
4. Using Materials Correctly

20	40	60	20	40	60	20	40	60

1. Following Script
2. Following Sequence
3. Fluent Pacing
4. Using Materials Correctly

20	40	60	20	40	60	20	40	60

1. Following Script
2. Following Sequence
3. Fluent Pacing
4. Using Materials Correctly

20	40	60	20	40	60	20	40	60

APPENDIX M
TEACHING SCRIPT FOR INSTRUCTION

Concrete to Abstract Lessons (Three-Level)

Lesson One: Identifying Ones and Tens Concretely With Cubes

Materials: plastic cubes, place value cards, vis-a-vis pen, probe, chart

Advance Organizer

"Today we're going to practice counting ones and tens using cubes. This will help us understand the meaning of two-digit numbers such as 15, 24, 32, 67." (Write these sample numbers on blackboard as they're said.) "Notice these numbers have two digits. One digit is the ones (point to the ones) and the other is the tens (point to the tens). Before we start let me show you the materials we'll use. These are cubes. What are they? . . . Yes, cubes. These are place value cards. What are they? . . . Yes, place value cards. Good! Let's begin."

Demonstrate/Model

(Have three groups of cubes on the table; one group of 23, one of 15, and one of 32.) "I'm going to count the first group of cubes and put them in groups of tens and

ones." (Count 23 cubes and then group them in tens and ones. Interlock the cubes that represent tens. Leave the ones cubes unconnected.) "Now I'm going to write how many tens I counted. How many tens? . . . Yes, I counted two tens." (Write the 2 on place value chart.) "How many ones do I have? . . . Yes, I counted three ones." (Write 3 on the place value chart.) "Another way to write this number is 23." (Write 23 on blackboard.) "This number is twenty-three. The number on the far right is in the ones place. The number two places over (count and point) is in the tens place. What number is on the far right? Yes, the ones. What number is two places over? Yes, the tens." (Repeat this procedure demonstrating 15 and 32.)

Guided Practice

(Give the student 35 cubes.) "You count and put the cubes in groups of tens and ones." (Teacher and student each count 35 cubes and group them by tens and ones.) "How many tens did we count? . . . We counted three tens. Write 3 in the tens column on your place value chart. . . . How many ones did we count? We counted five ones. Write 5 in the ones column. . . . Another way to write the 3 tens and 5 ones is 35. Write 35 on the other side of your card. Where is the ones place? Yes, on the far right. Where is the tens place? Yes, two places over." (Repeat this procedure with the numbers 43, 19, and 20.)

Independent Practice

(Give the student 26 cubes.) "Now you count and put the cubes in groups of tens and ones. Remember to record the number on your place value card." (Have students repeat independent practice with the numbers 72, 10, and 12.)

Criterion

Four out of four or 100% performed correctly. Repeat practice until criterion is met.

Administer 1-minute probe. Chart performance.

Lesson Two: Identifying Tens and Ones Concretely With Sticks

Materials: place value sticks, place value cards, probe, chart

Advance Organizer

"Yesterday we practiced counting tens and ones in double digit numbers using cubes. Remember? . . . You did a good job counting ones and tens and then writing the number on your place value cards. Today we're going to practice counting tens and ones using place value sticks. This will help us understand the meaning of two digit numbers such as 20, 89, 13, 51." (Write these sample numbers on the blackboard as they're said.) "Notice these

numbers have two digits. One digit is in the ones place (point to the ones) and the other is in the tens place (point to the tens). Before we start let me show you the materials we'll use. These are place value sticks. What are they? . . . Yes, place value sticks. Do you remember what these are?" (Hold up place value cards.) "Yes, place value cards. Good remembering. Today the double digit numbers are already written on the cards. Let's begin."

Demonstrate/Model

(Place cards in a stack with the double digit numbers upward.) "I'm going to pick a number card. It has a double digit number on it." (Show student the card.) "Now I'm going to use the place value sticks to show tens and ones in the number." (Count total number of sticks needed. Then count tens and hold together with rubberbands. Next, count the ones.) "I can check myself by turning the place value card over. . . . Yes, I have ___ tens and ___ ones. I was right."

Guided Practice

"You pick a card with a double digit number on it. We'll show with place value sticks the number of tens and ones in the number. Go ahead and count and group the sticks. . . . How many groups of ten do we have? . . . Is that the number in the tens place? . . . How many ones do

we have? . . . Is that the number in the ones place?
 . . . Turn your place value card over and check your
 sticks. . . . Did you get it right? . . . Yes, you did a
 good job. Let's try another one." (Repeat the procedure
 with another number card.)

Independent Practice

(Write a double digit number less than 70 on the
 blackboard.) "Show the number of tens and ones in this
 number with your place value sticks." (Check student
 answer and give feedback. Have student continue
 independent practice with self-correcting place value cards
 containing the numbers 70, 25, and 67.)

Criterion

Four out of four or 100% performed correctly. Repeat
 practice until criterion is met.

Administer 1-minute probe. Chart performance.

Lesson Three: Identifying Tens and Ones Concretely With Strips

Materials: place value strips, blank place value cards,
 place value cards with numbers, probe, chart

Advance Organizer

"We've already practiced counting tens and ones in double digit numbers using cubes and sticks. Today we're going to practice counting tens and ones using place value strips. By the end of this lesson you'll know how to count tens and ones three different ways. That's terrific. I'm proud of the work you've been doing. Keep it up. Let me show you the materials we're going to use today." (Hold up place value strips.) "These are place value strips. What are they? . . . Yes, place value strips." (Hold up a ten strip.) "Let's count the squares. Ready? Let's begin. One, two, three . . . ten. How many squares? . . . Yes, ten." (Hold up the strip.) "So this is one ten." (Hold up 3 ten strips.) "This is one, two, three tens. How many tens? . . . Yes, three tens." (Hold up place value card.) "What are these? . . . Yes, place value cards. Good. We're ready to begin."

Demonstrate/Model

"I'm going to count the squares and record the number. Each whole strip has 10 squares." (Pick up four whole strips and three individual squares.) "Watch me count. One, two, three, four tens and there are one, two, three ones. Now I'll write the number on my place value card. I have four tens and three ones." (Write 4 in the tens column and 3 in the ones column.) "The name for four

tens and three ones is 43." (Write 43 on back of place value card. Repeat the procedure demonstrating the numbers 13, 86, and 22).

Guided Practice

"I'll pick up the strips and you count." (Pick up five strips one at a time as student counts by tens.) "How many tens? . . . We have five tens." (Pick up six individual squares as student counts.) "How many ones? . . . We have six ones. What's the name for five tens and six ones? . . . That's right, it's 56. Write it. . . . How many tens? . . . Yes, five. How many ones? . . . Yes, six. Good job!" (Repeat this procedure demonstrating the numbers 41, 91, and 40.)

Independent Practice

(Present the student with 7 ten strips and 4 ones. Have student count and write the number. Have student repeat this practice with 3 ten strips and no ones. Give student place value cards with numbers and have him count the appropriate number of strips to represent 52, 21, and 10.)

Criterion

Four out of five or 80% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Four: Identifying Tens and Ones Semiconcretely With Sticks

Materials: four worksheets, probe, chart

Part One

Advance Organizer

"I'm very proud of the work you've been doing with place value. You can show me tens and ones using plastic cubes (hold up cube), place value sticks (hold up stick), and place value strips (hold up strip). That's very good. Today we're going to practice counting pictures of place value sticks. We'll be looking for tens and ones. We'll complete four worksheets together and then take our 1-minute timing." (Distribute worksheet number one.) "Put your finger on the bear. . . . Good following directions. This bear's name is Super 10. He's named Super 10 because he loves bundles of tens. Look at the flag he's carrying. How many bundles of tens are on the flag? . . . Yes, there are two. So how many tens are there? . . . Yes, two. And how many ones are there? . . . That's right, zero. So two tens and zero ones is written like this." (Point to the number 20.) "Two tens and zero ones is another name for? . . . Yes, 20. Very good. Now look at my sheet." (Point to one bundle of

ten.) "How many tens? Yes, one. How many ones left over? . . . Right, there are no ones. So one bundle of ten and zero ones is another name for? . . . Yes, ten." (Point to the number ten. Repeat this oral practice for the numbers 20 through 90.) "You did a super job. Now we're ready to begin our worksheet."

Demonstrate/Model

"Look at my sheet. I'm going to count the number of tens." (Point as you count.) "One, two, three, four. So, I write four next to the word tens like this." (Trace the four.) "There are four tens. Since there are no ones to count, I know the number is (trace the 40) four tens and zero ones or 40. So, first I counted and wrote the number of tens. Then I wrote the other name which includes tens and ones."

Guided Practice

"Now you count the bundles of ten. How many? . . . Yes, four. Trace the four next to the word tens. So what is the other name? . . . Yes, forty. Trace the four tens and zero ones. Good job. Let's do the next one together. How many tens? . . . Yes, two. Write two next to the word tens. The other name for two tens is two tens and zero ones or 20. Write this other name in the blank. . . . Good work! Now I'd like you to finish this

page and worksheet number two by yourself. When you finish put your pencil down, so I'll know you're ready to go on."

Independent Practice

(Distribute worksheet number two.) "The problems on this sheet are done the same way. Remember to do your best work."

Part Two

Advance Organizer

"You did a good job working by yourself. Now we're ready to go on to worksheet number three." (Distribute worksheet three.) "We're going to practice counting tens and ones. Put your finger on the first bear picture. . . . Good following directions. Super 10 bear has alot of sticks. He's counting groups of tens. So far he has two tens. Now look at the next picture. We see he counted three groups of ten and didn't have enough sticks to make another ten. So, how many tens? . . . Yes, three. And how many ones left over? . . . Yes, five." (Point to numbers.) "So he wrote three under the tens and five under the ones. Where is the ones place in this number? . . . Yes, on the far right. And where is the tens place? . . . Yes, two places over." (Count and point to the tens place.) "Let's see if we can count like Super 10 bear."

Demonstrate/Model

"Look at the first box. How many bundles of ten are there?" (Count and point to each bundle.) "There are four bundles of ten. Because there are four tens, I'm going to write a four in the tens column. How many ones are there? There are two ones. I'll write two in the ones column."

Guided Practice

"You do the next box. Count the bundles of tens and write the number in the tens column. Count the number of ones and write the number in the ones column. I'll watch and help."

Independent Practice

"You've got the hang of it now. So, go ahead and finish this page and worksheet four on your own."

(Distribute worksheet four.)

Criterion

On part one, 12 out of 14 or 86% performed correctly. On part two, 11 out of 13 or 85% performed correctly. Repeat practice until criterion is met on both parts of the lesson.

Administer 1-minute probe. Chart performance.

Lesson Five: Identifying Tens and Ones Semiconcretely With Cubes

Materials: five worksheets, probe, chart

Advance Organizer

"Yesterday we practiced counting place value sticks in drawings. We counted tens and ones and wrote the numbers they represent." (Hold up sample.) "Today we're going to count tens and ones using pictures of cubes. We'll complete five worksheets and then take our 1-minute timing."

Demonstrate/Model

(Hold up worksheet number one.) "This student has some cubes. She's going to see how many tens she has and how many ones." (Count the ten cubes.) "She has one ten and zero ones or 10. Watch me do the first one. I count one ten and write one next to the word ten. I count one one and write one next to the word ones. So, one ten and one one makes the number eleven. Watch me write it in the blank."

Guided Practice

"Now you do the next one. Count and write the number of tens. Count and write the number of ones. Then write the double digit number it makes. I'll watch and help. . . . Good job. You'll finish this worksheet in a

few minutes, but first I want to show you worksheets two and three." (Distribute worksheets two and three.) "These problems are done the same way. You count the tens and ones. The number of tens is written in the tens column like this." (Trace the two.) "And the number of ones is written in the ones column like this." (Trace the eight.) "What number does that make? . . . Right, 28. So I write two tens and eight ones or 28 in the blank. Do the next one by yourself. . . . Good job. Now I'd like you to complete these three worksheets. Put your pencil down when you're finished, so I'll know you're ready to go on."

Part Two

Advance Organizer

"You did a good job working by yourself. Our last two sheets involve counting tens and ones again. This time, we don't have to write the number of tens and then the number of ones. Instead we write the number the cubes represent. I'll show you what I mean."

Demonstrate/Model

(Hold up worksheet number four.) "In the first problem I see one ten and zero ones, so I write the number ten in the blank space. Ten means one ten and zero ones. I'll do another one." (Hold up worksheet number five.)

"First I'll count the tens. One, two, three, four, five, six and I'll write the six in the tens place; then I'll count the ones. One, and I'll write the one in the ones place. So, six tens and one one makes 61."

Guided Practice

"You do the next one. Count the tens and ones, and write the number the cubes represent. I'll watch and help."

Independent Practice

"Do you understand what you're supposed to do on these two sheets? O.K. You may begin."

Criterion

On part one, 16 out of 20 or 80% performed correctly. On part two, 16 out of 19 or 84% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Six: Identifying Tens and Ones Semiconcretely With Multiple Embodiments

Materials: five worksheets, probe, chart

Part One

Advance Organizer

"I'm very pleased with how quickly you've learned to count tens and ones using pictures of place value sticks and cubes. Today we're going to continue our practice. This time we'll be counting pictures of different objects, such as bananas, pencils, spools of thread, books, stars, hearts, etc. We have five worksheets and then our 1-minute timing. O.K. Let's begin."

Demonstrate/Model

(Distribute worksheets one, two, and three.) "Look at the first item on worksheet number one. How many groups of ten bananas are there?" (Count and point to each group.) "There are two groups of ten. Because there are two tens I'm going to write a two in the tens column. How many ones are there? There are three ones. I'll write three in the ones column."

Guided Practice

"You do the next one. Count and write the number of logs you see. I'll watch. . . . Good job. Now let's try one from worksheet number two. Look at the pencils and see if 32 is correct. . . . Is it? Yes, it is. Now do the next one yourself. What is your answer? . . . Yes, you

should have written 13 since there is one ten and three ones."

Independent Practice

"Now I'd like you to finish worksheets one, two, and three by yourself. Count carefully and do your best work."

Part Two

Advance Organizer

"Our next job is a little different. We're still counting tens and ones, but this time we get to make the groups of ten." (Distribute worksheets four and five.)

Demonstrate/Model

"Look at the hearts and watch me count." (Point and count the ten hearts circled.) "Since this makes ten I can circle it. There aren't enough hearts to make another ten, so I can't circle any more. Now I can record the number. One ten and three ones makes 13."

Guided Practice

"You do the leaves. First circle the groups of ten and then record the correct answer. I'll help if you have trouble. . . . Very good."

Independent Practice

Now you're ready to do the rest by yourself. Take your time and do your best work. When you finish we'll do our 1-minute timing."

Criterion

On part one, 21 out of 26 or 81% performed correctly. On part two, 15 out of 18 or 83% performed correctly.

Administer 1-minute timing. Chart performance.

Lesson Seven: Identifying Tens and Ones Abstractly With Semiconcrete Cues

Materials: two worksheets, blank paper, probe, chart

Advance Organizer

"Your place value work is coming along nicely. Today we're going to do two place value worksheets. Then we'll play a game and do our 1-minute timing."

Demonstrate/Model

(Hold up worksheet one.) "Look at item one. I'm going to write and loop the number of tens and ones in 27. How many tens are in 27? There are two tens. I'll write two in the tens column and loop two groups of ten

tubes. There are seven ones. I'll write seven in the ones column and loop seven tubes." (Distribute worksheets one and two.)

Guided Practice

"You do item two. Write and loop the number of tens and ones in 33. I'll watch and help if needed. Good job. Where is the ones place? . . . Yes, on the far right. Where is the tens place? . . . Yes, two places over."

Independent Practice

"Now you write and loop the rest of the problems on these two worksheets. . . . You did a good job of figuring those out by yourself. Now we can play a game. Listen to the rules. First you're going to write a double digit number. Then it's my job to draw pictures to illustrate the number. If correct on the first try, I get a point. If incorrect, you get a point. Then I write a number for you to illustrate. Whoever has the most points at the end of time, wins."

Criterion

Twelve out of 14 or 86% performed correctly on the worksheet and 80% of the student problems during the game must be correct on the first try.

Administer 1-minute probe. Chart performance.

Lesson Eight: Identifying Tens and Ones Abstractly With Worksheet and Cards

Materials: place value cards, vis-a-vis pen, worksheet, probe, chart

Advance Organizer

"You've done so well with counting tens and ones using pictures that you're now ready to do place value without pictures. After today's lesson you'll understand tens and ones without pictures. That's great. After we practice place value with numbers only, we'll do our 1-minute timing."

Demonstrate/Model

(Hold up place value card showing 56.) "This number is 56. In the number 56, the number two places over stands for tens. So there are five tens in this number. The number on the far right stands for ones. So there are six ones in this number. I'll write the tens in the ten column and the ones in the one column on the opposite side of the card." (After writing in the numbers, give the student a stack of place value cards.)

Guided Practice

"You look at the number and decide how many tens and ones there are in the number. Then turn your card over and write your answer in the correct columns. I'll watch and help if needed. . . . Good job."

Independent Practice

"Now complete the stack by yourself. When you've finished practicing with the cards, I'd like you to complete this worksheet." (Distribute worksheet.) "Do your best work on both the cards and the worksheet."

Criterion

On the place value cards, the student must get 16 out of 20 or 80% performed correctly. On the worksheet the student must get 12 out of 15 or 80% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Nine: Identifying Tens and Ones Abstractly With Worksheets

Materials: blank paper, practice sheet, worksheet, probe, chart

Advance Organizer

"Today we're going to review what you've learned about place value. We'll practice identifying tens and ones

in double digit numbers and then perform the 1-minute timing."

Demonstrate/Model

"Remember the number on the far right is in the ones place and the number two places over is in the tens place." (Write any double digit number on a blank sheet of paper.) "How many ones are in this number? . . . There are __ ones. How many tens? . . . There are __ tens. There are __ tens and __ ones in __." (Repeat this with two other double digit numbers.)

Guided Practice

(Distribute practice sheet. Begin with number one and randomly ask how many ones and/or tens there are in each number on the sheet. Then have the student write double digit numbers on a blank sheet and let him ask you how many tens or ones.)

Independent Practice

(Distribute the worksheet.) "Now you're going to do these by yourself. Look at the underlined digit in each number and write tens if it's in the tens column and ones if its in the ones column."

Criterion

Sixteen out of 20 problems or 80% performed correctly.

Administer 1-minute timing. Chart performance.

Abstract Lessons (One-Level)

Lesson One: Identifying Tens and Ones Abstractly With Place Value Cards and Worksheet

Materials: place value cards, vis-a-vis pens, worksheet, probe, chart

Advance Organizer

"Today we are going to begin a unit on place value. We're going to learn about tens and ones in double digit numbers. Understanding tens and ones will help us do a better job adding and subtracting big numbers. In today's lesson we'll be using place value cards (hold one up), vis-a-vis pens (hold one up), and a worksheet (show student the worksheet). At the end of our lesson we'll do a 1-minute timing."

Demonstrate/Model

(Hold up place value card showing 56.) "This number is 56. In the number 56, the number on the far right stands for ones. So there are six ones in this number. The number two places over stands for tens. So there are

five tens in this number. I'll write the tens in the ten column and the ones in the one column on the opposite side of the card. Where is the ones place in a number? . . . Yes, on the far right. Where is the tens place? . . . Yes, two places over." (Point and count. Give the student the stack of place value cards.)

Guided Practice

"You look at the number and decide how many tens and ones there are in the number. Then turn your card over and write your answer in the correct columns. I'll watch and help if needed. . . . Good job."

Independent Practice

"Now complete the stack by yourself. When you've finished practicing with the cards, I'd like you to complete this worksheet." (Distribute worksheet.) "Do your best work on both the cards and the worksheet."

Criterion

On the place value cards, the student must get 16 out of 20 or 80% performed correctly. On the worksheet the student must get 12 out of 15 or 80% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Two: Identifying Tens and ones Abstractly With Worksheet

Materials: worksheet, probe, chart

Advance Organizer

"You did a good job on yesterday's place value lesson. Today we're going to practice some more numbers using a different worksheet." (Write a double digit number on the blackboard.) "Who remembers which number represents the ones? Yes, the one on the far right. And which number represents the tens? Yes, the one two places over from the ones. Very good. You sound ready to begin."

Demonstrate/Model

(Distribute worksheet.) "Look at my sheet. I'm going to write the number of tens and ones in 23. How many tens are in 23? There are two tens. I'll write two in the tens column. How many ones in 23? There are three ones. I'll write three in the ones column."

Guided Practice

"Point to item number two. . . . Good following directions. You do item number two. Write the number of tens and ones in 27. I'll watch and help if needed."

Independent Practice

"Now you write the rest of the answers by yourself. Take your time and do your best work. When you finish, put your pencil down so I'll know you're ready to check your work." (When the student finishes the worksheet have him tell you the answer to each item orally. Praise correct answers and provide corrective feedback as needed.)

Criterion

Twenty-four out of 30 or 80% performed correctly.

Administer 1-minute probe. Chart Performance.

Lesson Three: Identifying Tens and Ones Abstractly Through Auditory Practice

Materials: paper, probe, chart

Advance Organizer

"Today we're going to practice our tens and ones a little differently. We're going to hear a double digit number, write the number, and then identify the tens and ones. Listening is very important today. Are you ready? Good, let's begin."

Demonstrate/Model

"You're going to be the teacher. Think of a double digit number and call it out to me." (After student says a double digit, say. . . .) "O.K. that's a good one. Now it's my turn to write the number you said on my paper. Next to the number I'm going to make a place value chart and fill in the number of tens and ones." (Make chart.) "There are ___ tens and ___ ones in the number ___."

Guided Practice

"Now it's your turn to be the student and I'll call out a number. Write the number 25. Then make a place value chart showing the tens and ones. . . . How many tens? . . . Yes, two. How many ones? . . . Yes, five. What's the number? . . . Yes, very good the number is 25. Now it's your turn to be the teacher again. Call a number out to me." (Repeat this procedure at least four times.)

Independent Practice

"I think you're doing very well. Now I'm going to call out ten double digit numbers and I want you to write them and make the place value charts by yourself. Ready? O.K. Let's begin." (Call the following numbers 12, 27, 43, 10, 35, 62, 22, 51, 19, 24.)

Criterion

Eight out of 10 or 80% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Four: Identifying Tens and Ones Abstractly With
Place Value Cards

Materials: place value cards, paper, probe, chart

Advance Organizer

"You're making very good progress with your place value skills. I'm really pleased. Today we're going to practice using our place value cards. Then we'll do our 1-minute timing."

Demonstrate/Model

"I've got my place value cards in a stack with the double digit numbers face up. I'm going to pick the first card up, say the number, and then write the number of tens and ones on my paper like this." (Demonstrate.) "Then I'm going to turn the card over and check my answer." (Turn card over and show student.) "Was I right? . . . Yes the number ___ has ___ tens and ___ ones."

Guided Practice

(Distribute deck of place value cards to student.)

"Now you get your deck of place value cards ready. Be sure the double digit numbers are all face up. Pick the first card, say the number, write the number of tens and ones on your paper, then check your answer. . . . Good, that was right. Where is the ones place? . . . Yes, the number on the far right. Where is the tens place? . . . Yes, two places over." (Point and count.)

Independent Practice

"Now go ahead and do the other cards the same way. Remember to check your answer each time."

Criterion

Twenty-four out of 29 or 83% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Five: Identifying Tens and Ones Abstractly to Reinforce Newly Acquired Skills

Materials: blank paper, practice sheet, worksheet, probe, chart

Advance Organizer

"Today we're going to review what you've learned about place value. We'll practice identifying tens and ones

in double digit numbers and then perform the 1-minute timing."

Demonstrate/Model

"Remember the number two places over is the tens and the number on the far right is the ones." (Write any double digit number on a blank sheet of paper.) "How many ones are in this number? . . . There are __ ones. How many tens? . . . There are __ tens. There are __ tens and __ ones in __." (Repeat this with two other double digit numbers.)

Guided Practice

(Distribute practice sheet. Begin with number one and randomly ask how many ones and/or tens there are in each number on the sheet. Then have the student write double digit numbers on a blank sheet and let him ask you how many tens or ones.)

Independent Practice

(Distribute the worksheet.) "Now you're going to do these by yourself. Look at the underlined digit in each number and write tens if it's in the tens column and ones if it's in the ones column."

Criterion

Sixteen out of 20 problems or 80% performed correctly.

Administer 1-minute timing. Chart performance.

Lesson Six: Identifying Tens and Ones Abstractly Using a Game Format

Materials: deck of game cards, score sheets, probe, chart

Advance Organizer

"Today we're going to play a game to practice place value. Listen carefully to the rules and then we'll begin. At the end of our time we'll do our 1-minute timing."

Demonstrate/Model

"See these cards? They each have a double digit number with one of the numbers underlined. To play the game the first player draws the top card and has to say whether the underlined number is in the ones or tens place. If correct on the first try, that player earns two points. If incorrect, the player earns zero points. Whoever has the most points at the end of the game wins. So, if it were my turn this is what I'd do." (Draw a card and give the correct answer. Then write score down.) "Do

you have any questions? O.K. Let's begin." (Place card back in the pile.)

Guided Practice

(Play game and assist student when he answers incorrectly. Provide immediate feedback. Intermittently praise correct answers.)

Independent Practice

(Several minutes before time runs out have student add his score to determine the winner. Then have the student practice the deck one time without awarding points. Keep track of errors.)

Criterion

Sixteen out of 20 or 80% performed correctly when practicing the deck of cards without game format.

Adminster 1-minute probe. Chart performance.

Lesson Seven: Identifying Tens and Ones Abstractly With Worksheets Providing Visual Practice

Materials: two worksheets, probe, chart

Advance Organizer

"You're doing a terrific job with identifying tens and ones. You earned a lot of points playing our game yesterday. Today we have two worksheets and our 1-minute timing to do. Let's see if you can keep up the good work."

Demonstrate/Model

"Look at my worksheet. The first item says three tens and two ones. I'm going to make a place value chart with the tens column on the left and the ones column on the right. I'll write three in the tens column and two in the ones column like this." (Demonstrate.) "Then I'll write the other name for two tens and three ones, which is 23, in the blank space. Where is the ones place? . . . Yes, on the far right. Where is the tens place? . . . Yes, two places over." (Point and count. Then distribute worksheets.)

Guided Practice

"Now you do item number two the same way. Make the chart, write the numbers in the correct columns and then write the other name for the number in the blank space. . . . Good job. You did that right. You'll finish this worksheet by yourself in just a minute, but first let's look at the second worksheet. This sheet has ten place value charts but no numbers. It's your job to think

of 10 double digit numbers, write the numbers and then fill in the charts. Let's do the first one together. Write any double digit number next to number one. . . . Good. Now fill in the place value chart. . . . Perfect."

Independent Practice

"Now go ahead and complete both worksheets on your own. Do your best work."

Criterion

Sixteen out of 20 or 80% performed correctly.

Administer 1-minute probe. Chart performance.

Lesson Eight: Identifying Tens and Ones Abstractly Through Auditory Practice

Materials: scrap paper for student to use if needed

Advance Organizer

"Today's lesson is really going to test your understanding of tens and ones. You're going to make numbers after I tell you the number of ones and tens. The order will be mixed up, so you need to pay close attention. Let me show you what I mean."

Demonstrate/Model

"Sometimes I'm going to start with ones and sometimes I'm going to start with tens. After I tell you how many of both I have, your job is to come up with the right number. For example, if I say three ones and two tens, you must say 23."

Guided Practice

"Let's try a few together before I start counting corrects and errors. Ready? Here's the first one. Two tens and seven ones. What number? . . . Yes, 27. Very good. Here's another, five ones and one ten. What number? . . . Yes, 15." (Repeat this practice with four other examples.)

Independent Practice

"O.K., now we're warmed up and ready to begin." (Call these out to students and keep track of correct and error responses.)

- | | | |
|----|------------|------------|
| 1. | five tens | three ones |
| 2. | two ones | six tens |
| 3. | eight tens | one one |
| 4. | one ten | one one |
| 5. | six ones | two tens |
| 6. | three ones | four tens |
| 7. | one ten | five ones |

- | | | |
|-----|------------|------------|
| 8. | five ones | six tens |
| 9. | nine ones | eight tens |
| 10. | four tens | nine ones |
| 11. | three tens | two ones |
| 12. | two ones | four tens |
| 13. | four tens | nine ones |
| 14. | five ones | five tens |
| 15. | seven ones | seven tens |

Criterion

Twelve out of 15 or 80% correctly performed.

Administer 1-minute probe. Chart performance.

Lesson Nine: Identifying Tens and Ones Abstractly: A Summary

Materials: place value cards, vis-a-vis pen, probe, chart

Advance Organizer

"You've done very good on your place value unit. Today we're going to review all that you've learned. Then you'll be ready for your test."

Demonstrate/Model

(Hold up place value card showing double digit number.) "We learned that the number on the far right is the ones and the number two places over is the tens. What number is on the right? . . . Yes, ones. What number is two places over? . . . Yes, tens. So in this number the ___ is in the tens place and the ___ is in the ones place."

Guided Practice

"I'm going to flash some number cards. You tell me how many tens and how many ones there are. Ready?" (Flash ten cards to practice this.)

Independent Practice

"Now I'm going to give you some cards to do independently. Your job is to look at the number, then turn the card over and write how many tens and ones are in the number." (Give student the cards to complete. When the student finishes, check his work and then practice orally using the same cards. Show the student the double digit number and randomly ask him how many tens or ones are in the number.)

Criterion

Twenty-eight out of 34 or 82% performed correctly on written portion of lesson. Eight out of 10 or 80% performed correctly on oral portion of lesson.

Administer 1-minute probe. Chart performance.

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BIOGRAPHICAL SKETCH

Susan K. Peterson was born on January 12, 1956, in Cheyenne, Wyoming. Prior to the age of six she moved with her family to Massachusetts, Florida, and California. In 1961, her family moved back to Florida and settled in Orlando.

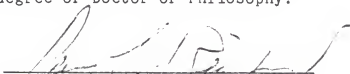
Susan graduated from Colonial High School and then attended Florida Southern College in Lakeland. In 1978, she earned her B.S. degree in sociology with a teaching certificate in secondary social studies. Susan began part-time graduate study in 1978 at the University of Florida and completed her M.Ed. in special education in 1981. She began part-time doctoral study in 1983. Her planned program included cognates in learning disabilities and emotional handicaps with a minor in educational administration.

Susan's work experiences include vocational counseling, teaching, and administration. In 1978, Susan was hired as a vocational counselor and employability skills instructor for economically disadvantaged adults enrolled in the Comprehensive Employment Training Program. Next, Susan worked for the Alachua County School

Board. She taught seventh, eighth, and ninth grade social studies to regular education students; compensatory math to ninth and tenth graders; and reading and language arts to sixth, seventh, and eighth grade students in a varying exceptionalities program.

In 1981, Susan was hired as the Lead Teacher for the University of Florida Multidisciplinary Diagnostic and Training Program (MDTP) which serves kindergarten through sixth grade students experiencing learning, behavioral, or medical problems. After 4 years teaching in the MDTP Diagnostic Classroom, Susan was promoted to Program Manager and has worked in that capacity since 1985.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Cary L. Reichard, Chairman
Professor of Special Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



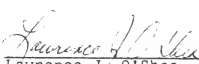
Phillip A. Clark
Professor of Educational
Leadership

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Myron A. Cunningham
Professor Emeritus of Special
Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Lawrence J. O'Shea
Assistant Professor of Special
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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